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Method of removing the domain to identify the position of the heat source by the inverse method

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Abstract: This research work is a numerical study which conducted a spatiotemporal identification of a thermal source inside a domain using inverse method. Temperatures are simulated by means of resolving direct problem with finite differences method. The authors suggested heat source identification in diffusive medium. The studied problem has two distinctive aspects: (1) source location research; (2) identifying source amplitude in terms of time. Direct simulation: The authors achieved a leveled scale on the studied source over the total duration of the study time interval. Step responses, at measuring points, are saved (for matrix building). After that, the authors computed source value at every step time. In order to characterize inverse quality, the authors have introduced mean square deviations corresponding to inverse process, we notice that we have a good result for the location, whilst the intensity of the source volume we have a worse outcome, because we identified a point source is equivalent to an approximate solution.

Keywords: heat source, inverse method, convolution integral, finite differences.

طريقة إزالة المجال لتحديد موضع مصدر الحرارة بالطريقة العكسية

عبد الكريم معمر عبد القادر
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المستخلص: هذا العمل البحثي عبارة عن دراسة رقمية أجريت تحديدًا مكانيًا زمانيًا لمصدر حراري داخل مجال باستخدام الطريقة العكسية. يتم محاكاة درجات الحرارة عن طريق حل مشكلة مباشرة بطريقة الفروق المحدودة. اقترح المؤلفون تحديد مصدر الحرارة في وسط منتشر. المشكلة المدروسة لها جانبان مميزان: (1) البحث عن موقع المصدر. (2) تحديد سعة المصدر من حيث الوقت. المحاكاة المباشرة: حقق المؤلفون مقياسًا مستويًا على المصدر المدروس خلال المدة الإجمالية للفاصل الزمني للدراسة. يتم حفظ استجابات الخطوة، عند نقاط القياس، (لبناء المصفوفة). بعد ذلك، قام المؤلفون بحساب قيمة المصدر في كل خطوة مرة. من أجل توصيف الجودة العكسية، أدخل المؤلفون متوسط الانحرافات التربيعية المقابلة للعملية العكسية. نلاحظ أن لدينا نتيجة جيدة للموقع، في حين أن شدة حجم المصدر لدينا نتيجة أسوأ، لأننا حددنا أن مصدر النقطة يعادل حلاً تقريبيًا.

الكلمات المفتاحية: مصدر الحرارة، الطريقة العكسية، التلافيف المتكامل، الفروق المحدودة.

I. Introduction

According to Keller [1], two problems are called inverse to each other if the formulation of one puts the other involved. This definition contains an element of arbitrariness, and plays a role symmetrical to the two problems considered. A more operational definition is that an inverse problem is to determine the causes experiencing effects. Thus, the problem is the reverse of the one called direct problem, of deducting the effects, the causes are known.

Data processing means enable us to reach a significant development in thermal systems numerical modeling. Determining temperature and flux fields is made through two types of factors:

- Parameters which are intrinsic to the studied system as, for instance, thermal properties;
- Parameters which are external to the system, and belong to thermal stresses as, for instance, boundary conditions, internal sources or heat sinks, and heat flux.

This research work is put in the framework of inverse methods consisting of determining thermal stresses by means of temperature measures (or measures simulation).

In many industrial applications, a direct measure of heat flux or surface temperature (inner wall of an engine combustion chamber or steam generators piping, interfaces between brakes pads and discs, external surface of a spacecraft penetrating the atmosphere, etc.) is complicated and sometimes impossible. Determining these surface factors, out of transitory temperature measures inside and on solid faces, is an inverse problem of heat conduction.

Although they have a practical interest, methods that are able of resolving inverse problems of heat transfer are very few in Refs. [2, 3].

We deal in this work a problem of identification of a volume heat source in a diffusive 2D, the source is assumed uniform. We start first by looking for its position is determined by its amplitude as a function of time.

To address this problem, we use a direct modeling by finite difference, to store the simulated temperatures. The other way round it is based on an inversion of the integral of convolution type Beck [1]. In this phase inversion of the system is considered under the aspect monoentrée (the source to be identified) multisorties (data from internal and surface temperatures). The source density 2D we use to simulate our temperature measurements is circular.

II. Inversion method

II.1. Integral Convolution

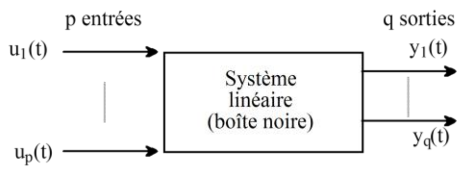

Fig (1) Principle of external representation

Knowing the matrix H (t), then it is conventional to write the system's response to any vector u (t) in the form of the convolution integral [2-10].

$$y(t)=\int_{t_0}^{t}H(t-\tau)u(\tau)d\tau \qquad (1)$$

where u (t) is the vector of input dimension (p), y (t) the output vector of dimension (q), H (t) the impulse response matrix of dimension (qxp), t is time, t_0 the initial time for which y (t_0) = 0. Using in equation (1) the matrix of responses index M (t) it comes:

$$y(t)=\int_{t_0}^{t}-\frac{dM(t-\tau)}{d\tau}u(T)d(T \qquad (2)$$

Δt is the constant time step. By putting t = FΔt l'instant calculation and t= f Δt current time integration, a finite difference approximation to the first order derivative of M (t), it comes:

$$y(F)=\sum_{f=1}^{F}\Delta M_{F-f}u(f)$$
$$\text{of} \quad \Delta M_{F-f}=M(F-f+1)-M(F-f) \qquad (3)$$

To determine the vector u entries F at the moment, we will use equation (3) extended to the future time step. Explain this equation by writing it at the time of calculating F:

$y(F) = \Delta M_{F-1} u(1) + \Delta M_{F-2} u(2)+....+ \Delta M_1 u(F-1)+ \Delta M_0 u(F)$ (4)

In this expression u (1),... u (F-1) are known, u (F) is unknown. We may combine in a term y * (F) any contribution of the moments prior to F in the form:

$y^*(F) = \Delta M_{F-1} u(1) + \Delta M_{F-2} u(2)+....+ \Delta M_1 u(F-1)$ (5)

Equation (4) daemon:

$y(F) - y^*(F) = \Delta M0\, u(F)$ (6)

Or in matrix form: $Y = M\, u(F)$ (7)

The relationship (7) constitutes a system of q * (R +1) equations with p unknowns. In general this system is over determined, because q * (R +1)> p. The exact resolution is not possible, by choosing a standard quadratic, it is then the solution of equation (7) in the sense of least squares, either:

u(F) = (MT M)-1 MT Y (8)

The inverse problem is solved at each time step F, taking into account R future time steps, by the relation (8).

II.2 The direct model finished in deference

The finite difference equations can be established in two ways, either by using the results of numerical analysis, or by writing the bilant thermal each node of the network. The second method allows a more physical problem [3], [7].

II.1.1. Explicit method

$$\Delta T - \frac{1}{a}\frac{\partial T}{\partial t} + \frac{P(M,t)}{\lambda} = 0 \qquad (9)$$

$$T_{i,j}^{t+\Delta t} = c_i T_{i,j} + a_i\left(T_{i+1,j} + T_{i-1,j}\right) + b_i\left(T_{i,j+1} + T_{i,j-1}\right) + P_{i,j}\frac{a\Delta t}{\lambda\Delta x\Delta y} \qquad (10)$$

$$T_{n,j}^{t+\Delta t} = T_{n,j}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda\Delta y}\right) + a_i\left(T_{n,j-1} + T_{n,j+1}\right) + 2b_i T_{n-1,j} + \frac{2ah\Delta t}{\lambda\Delta y}T_\infty \qquad (11)$$

$$T_{1,j}^{t+\Delta t} = T_{1,j}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda\Delta y}\right) + a_i\left(T_{1,j-1} + T_{1,j+1}\right) + 2b_i T_{2,j} + \frac{2ah\Delta t}{\lambda\Delta y}T_\infty \qquad (12)$$

$$T_{i,m}^{t+\Delta t} = T_{i,m}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda\Delta x}\right) + b_i\left(T_{i+1,m} + T_{i-1,m}\right) + 2a_i T_{i,m-1} + \frac{2ah\Delta t}{\lambda\Delta x}T_\infty + P_{1,m}\frac{2\Delta t}{\rho c\Delta x\Delta y} \qquad (13)$$

$$T_{i,1}^{t+\Delta t} = T_{i,1}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda\Delta x}\right) + b_i\left(T_{i+1,1} + T_{i-1,1}\right) + 2a_i T_{i,2} + \frac{2ah\Delta t}{\lambda\Delta x}T_\infty + P_{i,1}\frac{2\Delta t}{\rho c\Delta x\Delta y} \qquad (14)$$

$$T_{n,1}^{t+\Delta t} = T_{n,1}\left(1 - 2a_i - 2b_i - 2\frac{ha\Delta t(\Delta x + \Delta y)}{\lambda\Delta x\Delta y}\right) + 2a_i T_{n,2} + 2b_i T_{n-1,1} + 2\frac{ha\Delta t(\Delta x + \Delta y)T_\infty}{\lambda\Delta x\Delta y} \qquad (15)$$

$$T_{n,m}^{t+\Delta t} = T_{n,m}\left(1 - 2a_i - 2b_i - 2\frac{ha\Delta t(\Delta x + \Delta y)}{\lambda\Delta x\Delta y}\right) + 2a_i T_{n,m-1} + 2b_i T_{n-1,m} + 2\frac{ha\Delta t(\Delta x + \Delta y)T_\infty}{\lambda\Delta x\Delta y} \qquad (16)$$

$$T_{1,m}^{t+\Delta t}=T_{1,m}\left(1-2a_i-2b_i-2\frac{ha\Delta t(\Delta x+\Delta y)}{\lambda\Delta x\Delta y}\right)+ \quad (17)$$
$$2a_iT_{1,m-1}+2b_iT_{2,m}+2\frac{ha\Delta t(\Delta x+\Delta y)T_\infty}{\lambda\Delta x\Delta y}$$

$$T_{1,1}^{t+\Delta t}=T_{1,1}\left(1-2a_i-2b_i-2\frac{ha\Delta t(\Delta x+\Delta y)}{\lambda\Delta x\Delta y}\right)+$$
$$2a_iT_{1,2}+2b_iT_{2,1}+2\frac{ha\Delta t(\Delta x+\Delta y)T_\infty}{\lambda\Delta x\Delta y}+P_{1,1}\frac{4\Delta t}{\rho c\Delta x\Delta y} \quad (18)$$

With: $a_i=\frac{a\Delta t}{\Delta x^2}$, $b_i=\frac{a\Delta t}{\Delta y^2}$ et $c_i=1-\frac{2a\Delta t}{\Delta x^2}-\frac{2a\Delta t}{\Delta y^2}$

II.2.1. Implicit method

II.2.1.1. Principle of the method

Flows converge towards the node Mi, j are calculated from the values of temperatures at t and t +Δt [8].

$$\phi_{i,j}=(1-k)\phi_{i,j}^{-}+k\phi_{i,j}^{+} \quad (0<k\le 1) \quad (19)$$

II.2.1.2 Equations implicit finite difference

The conductive flux written:

$$\varphi_{i,j-1}=\frac{\lambda}{\Delta x}\left((1-k)(T_{i,j-1}-T_{i,j})+k(T_{i,j-1}^{+}-T_{i,j}^{+})\right) \quad 20)$$

$$\varphi_{i,j+1}=\frac{\lambda}{\Delta x}\left((1-k)(T_{i,j+1}-T_{i,j})+k(T_{i,j+1}^{+}-T_{i,j}^{+})\right) \quad (21)$$

$$\varphi_{i-1,j}=\frac{\lambda}{\Delta y}\left((1-k)(T_{i-1,j}-T_{i,j})+k(T_{i-1,j}^{+}-T_{i,j}^{+})\right) \quad (22)$$

$$\varphi_{i+1,j}=\frac{\lambda}{\Delta y}\left((1-k)(T_{i+1,j}-T_{i,j})+k(T_{i+1,j}^{+}-T_{i,j}^{+})\right) \quad (23)$$

With:

$H=C_i+2\Delta x^2+2\Delta y^2$ k=1, $ai=\frac{h\Delta x^2\Delta y}{\lambda}$ $bi=\frac{h\Delta y^2\Delta x}{\lambda}$ and $Ci=\frac{\Delta x^2\Delta y^2}{a\Delta t}$ $D=a_i+b_i+\Delta x^2+\Delta y^2$

- For an internal node:

$$CiT_{i,j}+P_{i,j}\frac{\Delta x\Delta y}{\lambda}=HT_{i,j}^{+}-\Delta y^2T_{i,j-1}^{+}-\Delta y^2T_{i,j+1}^{+}-\Delta x^2T_{i-1,j}^{+}-\Delta x^2T_{i+1,j}^{+} \quad (24)$$

- For a node located on a flat surface:

$$2aiT_\infty+CiT_{n,j}=(2ai+H)T_{n,j}^{+}-\Delta y^2T_{n,j-1}^{+}- \quad (25)$$
$$\Delta y^2T_{n,j+1}^{+}-2\Delta x^2T_{n-1,j}^{+}$$

$$2aiT_\infty+CiT_{1,j}=(2ai+H)T_{1,j}^{+}-\Delta y^2T_{1,j-1}^{+}- \quad (26)$$
$$\Delta y^2T_{1,j+1}^{+}-2\Delta x^2T_{2,j}^{+}$$

$$2biT_\infty + CiT_{i,1} = (2bi + H)T^+_{i,1} - \Delta x^2 T^+_{i-1,1} - \Delta x^2 T^+_{i+1,1} - 2\Delta y^2 T^+_{i,2} \quad (27)$$

$$2b_i T_\infty + C_i T_{i,m} = (H + 2b_i)T^+_{i,m} - \Delta x^2 (T^+_{i-1,m} - T^+_{i+1,m}) - 2\Delta y^2 T_{i,m-1} \quad (28)$$

- For a node located at an angle outside:

$$(ai + bi)T_\infty + 0.5CiT_{n,1} = (0.5Ci + D)T^+_{n,1} - \Delta y^2 T^+_{n,2} - \Delta x^2 T^+_{n-1,1} \quad (29)$$

$$(ai + bi)T_\infty + 0.5CiT_{n,m} = (0.5Ci + D)T^+_{n,m} - \Delta y^2 T^+_{n,m-1} - \Delta x^2 T_{n-1,m} \quad (30)$$

$$(ai + bi)T_\infty + 0.5CiT_{1,m} = (0.5Ci + D)T^+_{1,m} - \Delta y^2 T^+_{1,m-1} - \Delta x^2 T_{2,m} \quad (31)$$

$$(ai + bi)T_\infty + 0.5CiT_{1,1} = (0.5Ci + D)T^+_{1,1} - \Delta y^2 T^+_{1,2} - \Delta x^2 T^+_{2,1} \quad (32)$$

III. The system studied and its modeling

In Figure 3 are represented the various elements of the problem:

- Geometry studied is a flat rectangular plate of length 0.2 m and a width of 0.1 m. The thermo-physical characteristics are selected for a building material like plaster. Values [5]:
  - Thermal conductivity $\lambda = 1.5\ Wm^{-1}.k^{-1}$
  - Density $\rho = 2300\ kg.m^{-3}$
  - Heat capacity $c = 800\ J.kg^{-1}.k^{-1}$
- On all 4 sides of the rectangle, the boundary conditions of the field are Fourier type ($h=20\ W\ m^{-2}\ K^{-1}$) in relation to a reference temperature taken as 0 ° C. At the initial moment, the whole system is at 0 ° C [9].
- The circular source centered at point C: $xc = 5\ 10^{-2}$ m, $yc = 6\ 10^{-2}$ m and radius $R = 2.5\ 10^{-2}$ m.
- To model this system in deference over, you have a mesh of the plate $\Delta Y = 0005$ m and $\Delta X = 0.01$ m.

P (t) the power emitted by this source as a function of time (see Figure 4).

IV. Location method (method of removal of the domain)

The idea is based on an iterative method, presented by the following algorithm [4] (see Figure 2):

1- The plate (with dimensions: xl = 0.2 m, yl = 0.1 m) is separated into two (2) A and B at its center, by a parallel to Oy.

2- Two sources fictitious SP-A and SP-B are placed in each of the points A and B, the abscissa xa = 3 / 8 xl and xb = 5 / 8 xl and ya = yb = yg = 1 / 2 yl.

3- Recovery of simulated temperatures.

4- Calculation of index responses for each source (SP-A and SP-B) on the first step (construction of the matrix M of (3).

5- Calculation of the vector u (1) by the relation (8) on the first step which is:

u(1) = (MT.M)-1 MTY(R).

The vector u (1) has two components ua (1) and ub (1) with respect to SP-A and SP-B.

6- A center of gravity between SP-A and SP-B will be calculated.

7- The part not containing the center of gravity is "disposed" of the algorithm.

8- It works in the opposite direction on a parallel to Oy via xg creation of SP-A and SP-B following xa = xb = xg, where ya = 3 / 8 and yb= 5 / 8 yl. Reversing the x and y and on again in 2 until convergence (Fig 5).

9- The final solution, ie the position of the source, is the center of gravity when it almost does not move during the iterations.

Once the source is located on the first step, there is a direct simulation:

it makes a step on this source over the length of the interval time of study. Replies index, the measurement points, are stored (for the construction of the matrix M) is calculated by the source value at each time step, from the relationship (8).

V. Result

The location is therefore searching for a point source "equivalent" (SPE), successive iterations can identify the center of gravity G, the position taken by the PES, the "epicenter" of the source is located fairly accurately (figure 2).

In Figure 5, it represents the convergence of the position of G according to the number of iterations, you can see how successive calculations of xg and yg tend towards the values xc and yc sought. SPE is localized, then we apply the inversion method on the whole time horizon.

We present in Figure 6, the source value compared with the SPE theoretical curve used for the direct model and the quadratic differential on sources e_S (W / m). We note that levels are not well reproduced despite the best location, the reason that the two sources are not similar because here we identify a point source equivalent.

In Figure 7 presents examples of thermogram reconstructed from identification with the root mean square deviation and (in ° C), we see a good agreement until the time t = 4000s, after this time is a little less and a better fit for the curves of temperatures that are far from the source.

In thermics, the best criterion to justify a good choice of such a method is the quadratic deviation on the temperatures reconstructed from the identified curves and the simulated temperatures, we find e_T = 0.42°C.

VI. Conclusion

In this work, we presented a method of inversion in thermal conduction, to identify a volume heat source in a diffusive 2D. This inverse problem has two steps to locate the position of this source, then an identification of its amplitude as a function of time. So on the heat source density, we notice that we have a good result for the location, whilst the intensity of the source volume we have a worse outcome, because we identified a point source is equivalent to an approximate solution, but the final solution is still the identification of the geometric shape of this source.

This inverse problem includes two identifications in two-dimensional geometry:

- identification of the amplitude of the point heat source,
- identification of the heat source position.

For each of these points, we took care to put our algorithms to the test, with simulated temperatures. These come from a direct model, where all the thermal stresses and the parameters are perfectly known. These methods set up are simple concepts, the inverse problems being badly posed, we especially sought to set up robust algorithms to have a convergence towards a physically acceptable solution. With regard to the numerical methods and the algorithms retained, let us recall the main characteristics:

- for direct modeling: use of the formulation by the finite difference method.
- for the inverse modelization: concerning the identification of the amplitude of the heat source, one used on the one hand a method of the Beck type, involving future time steps, consisting of a convolution of the responses of the system on previously determined transfer functions. Different case studies are considered when solving inverse problems such as: number and location of sensors, time step, measurement noise, in order to analyze the behavior of the identification algorithm.

From the point of view of the operation of numerical simulation, the following aspects can be highlighted:

- the point source of heat: the temporal identification of the point source in 2D is very satisfactory, the intensity of this source is very well "found"
- The position of the heat source is well determined.

Finally, from the point of view of potential applications, we can cite the search for hot spots in tribology, the identification of source terms in electric motors, the search for thermal sources within a medium where an exothermic chemical reaction takes place.

Figures

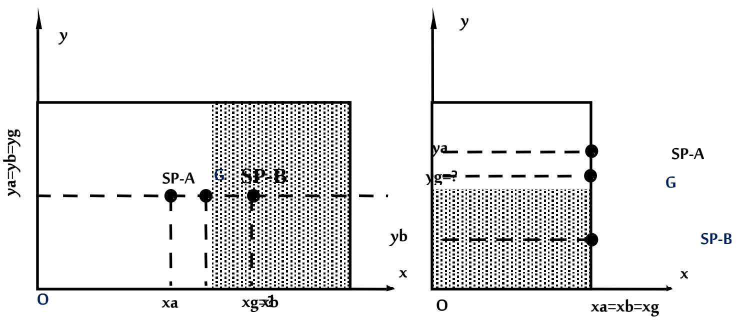

Fig (2) Principle of localization, elimination of field

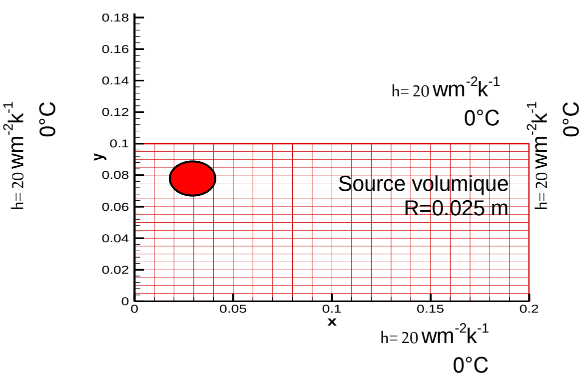

Fig (3) The test case studied with these conditions

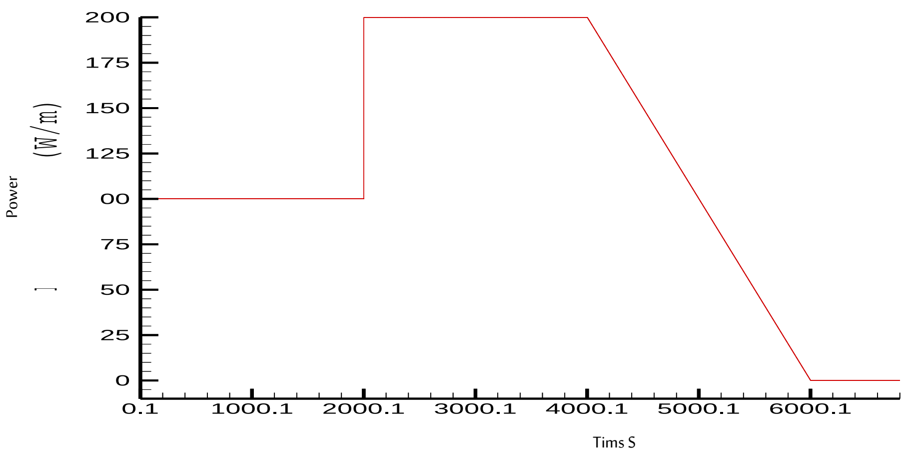

Fig (4) Evolution of power total P (t) of the circular source

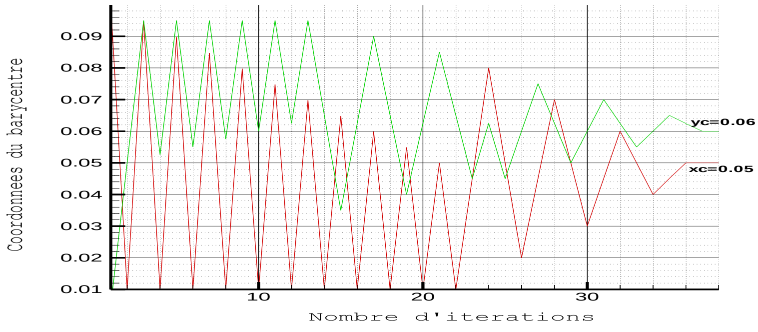

Fig (5) Convergence of center of gravity

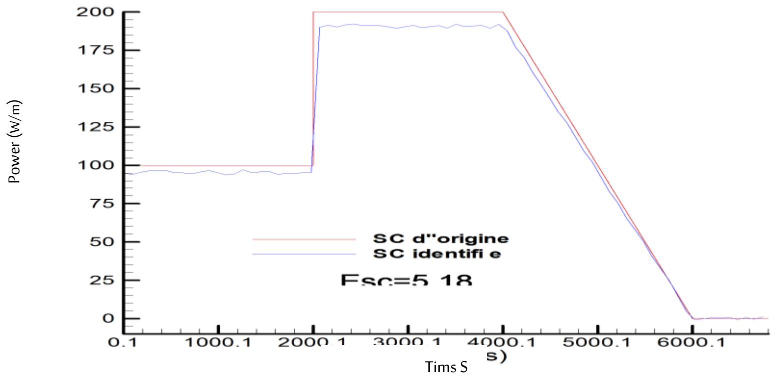

Fig (6) Comparison of the circular source of origin and the point source equivalent (SPE)

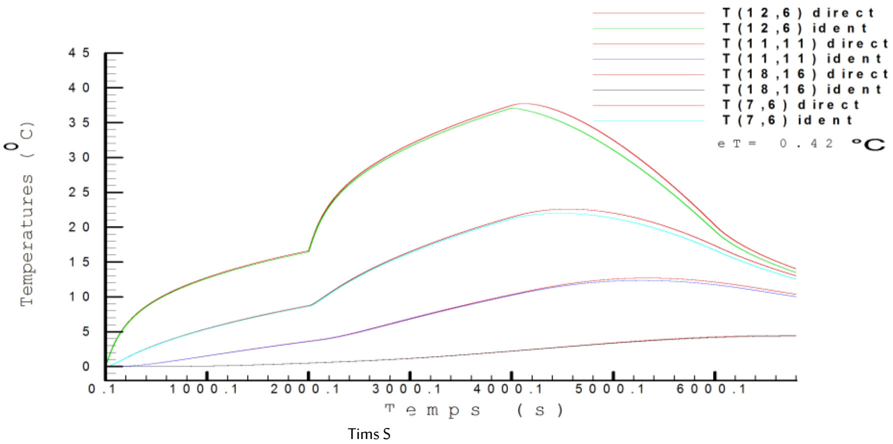

Fig (7) Evolution of internal temperatures comparison

Of Simulated and measures identified (SPE)

Nomenclature

a: thermal diffusivity, $m^2.s^{-1}$

c: specific heat, J. $kg^{-1}.K^{-1}$

eS, eT: standard deviation, ° C

F: final time, s

h: convection coefficient, $W.m^{-2}.K^{-1}$

H (t): impulse responses,

M (t): step responses,

t: time, s

T: temperature, K

u (t), y (t): vector

xc, yc: coordinates of the source, m

Greek symbols

Δ : Laplacian operator

λ : thermal conductivity, $W.m^{-1}.K^{-1}$

ρ : density, $kg.m^{-3}$

τ : time, s

Indices

EPS: point source equivalent

i, j, k, n, m: indices

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