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Simulating NACA Equations Used in Optimizing Wind Turbine Blade Design

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Abstract: Aerodynamic scientists are interested in geometry definition and possible geometric shapes that would be useful in design. This paper illustrates a simulation of a NACA four digits airfoil blade profile using MATLAB. As airfoil design became more sophisticated, this basic approach has been modified to include additional variables, and suggestions for the chord line length at the root and at the end of the blade. as well as changes in the twisting angle of the blade and its thickness, this helps to reduce the weight of the blade significantly Simulating NACA equations is very useful in obtaining coordinates of airfoil curvature for the whole series of NACA four digits, which is very effective in optimizing blade design. In order to get an optimal operating performance and high efficiency for the airfoil, the blade surface must be smooth and does not suffer any discontinuities or undefined cases, which cause separation of the boundary layer during the airflow, and get as a result great energy losses. Therefore, the conditions for the continuity of the blade was extracted using mathematical analysis, so the air flow does not suffer any interruptions which reduce the efficiency. This enable us to determine the locations of the maximum thickness of the blade sections on the chord along the blade, in addition to specifying conditions for the chord line length at the root and at the end of the blade which keep the blade curvature continuous and doesn't have any irregular points, which also facilities writing the necessary programs.

Keywords: Chord line, Mean line, Blade length, Maximum camber, Undefined case, Curvature discontinuities.

محاكاة معادلات NACA واستخدامها لتحسين تصميم شفرة العنفة الريحية

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المستخلص: يهتم علماء الأيروديناميك بتعريف الشكل الهندسي للشفرة والأشكال الهندسية المحتملة التي قد تكون مفيدة في التصميم. يوضح هذه البحث نمذجة لمعادلات NACA four digits باستخدام ماتلاب. وبما أن تصميم شفرة العنفة الريحية يعتبر معقدا، فقد تم تعديل هذه المنهجية لتشمل متغيرات إضافية، واقتراحات لطول خط الوتر عند الجذر وفي نهاية الشفرة وكذلك تغيرات زاوية فتل الشفرة وسماكتها كل ذلك يفيد في خفض وزن الشفرة بشكل معتبر. أن نمذجة معادلات NACA مفيد جدًا في الحصول على إحداثيات بروفايل اي شفرة في سلسلة NACA four digits، وهو أمر فعال للغاية في تحسين تصميم الشفرة.

من أجل الحصول على الأداء التشغيلي الأمثل والكفاءة العالية للعنفة الريحية، يجب أن يكون سطح الشفرة أملساً ولا يعاني أي انقطاعات أو حالات عدم تعيين، الذي يؤدي إلى انفصال الطبقة الحدية أثناء تدفق الهواء وهذا ما يسبب فقدان كبير في الطاقة. لذلك، في هذه الدراسة تمكنا عن طريق التحليل الرياضي من اضافة شرط استمرارية الشفرة لكي لا يعاني منحني الشفرة وبالتالي جريان الهواء

عليها من انقطاعات تؤدي إلى التقليل من مردود المنشأة، وهذا مكننا من تحديد مواقع السماكة الأعظمية لمقاطع الشفرة على الوتر وذلك على طول الشفرة، وعلاوة على ذلك تحديد شروط لقيم وتر الشفرة عند الجذر و عند النهاية بما يضمن المحافظة على استمرارية منحني الشفرة. أن اختيار الأبعاد ضمن هذه الشروط يؤدي إلى تجنب حدوث الانقطاعات وحالات عدم التعيين في البروفايل وهذا ما يضمن الحصول على انحناء أملس لسطح الشفرة وهذا ما يساعد أيضاً في تسهيل كتابة البرامج اللازمة.

الكلمات المفتاحية: خط الوتر، الخط الوسطي، طول الشفرة، التحدب الأعظمي، حالة عدم تعيين، انقطاعات في الانحناء.

1. Introduction

National advisory committee for aeronautics described airfoil geometry using digits after the word NACA. NACA Four- Digit Series (MPXX) the first digit specifies the maximum camber (M) in percentage of the chord (airfoil length), the second indicates the position of the maximum camber (P) in tenths of chord, and the last two numbers provide the maximum thickness (XX) of the airfoil in percentage of chord as shown in figure (1).[2]

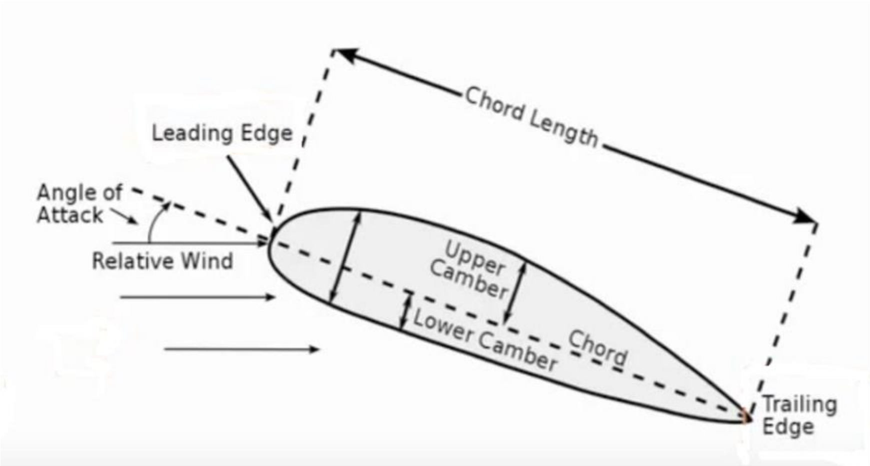


Figure (1) Blade parameters [2]

To get the optimal geometries and superior aerodynamic performance circle method has been modified and applied on the design of 2D isolated airfoils [5]. While in small wind turbine blade, it has been found that the fluent calculation of NACA63- 215V showed that discontinuities were so small that the blade performance could not be affected but there were difficulties to connect to Bezier curve, and programming difficulties [3]. While, in large size wind turbine blades, an absolute nodal coordinate formulation has been used for modelling and investigated in the methods of modelling slope discontinuity resulting from the variations of the cross- sectional layouts across the blade[1].

Some researchers have applied circle method to a symmetrical airfoil and a non- symmetrical airfoil, to remove their surface curvature and slope- of- curvature discontinuities [9]. Others have applied the prescribed surface curvature distribution blade design method on airfoil E387 to remove the gradient of curvature discontinuities [8]. In addition, two- dimensional direct blade- design method has been developed, which uses a mixture of analytic polynomials and a mapping of a desired curvature distribution on the (x, y) plane to specify the geometry and avoid small slope- of- curvature discontinuities in the blade surfaces [4]. A genetic algorithm have been used to optimize the airfoil shape considering a balance between the aerodynamic and structural performance of airfoils [10]. Moreover, PARSEC geometry method has been applied on the airfoil profile and represented its shape as a polynomial function [7].

A computational framework has been developed for shape optimization of wind turbine blades for variable operating conditions specified by local wind speed distributions [11]. Moreover, the effects of jet width on blowing and suction flow control has been evaluated and found that the jet widths of 3.5% and 4% of the chord length are the most effective amounts for tangential blowing, and smaller jet widths are more effective for perpendicular blowing [12].

We can summarize previous researchers' efforts in applying different approaches to get the optimal wind blade geometry, which lead to optimal performance.

This research aims to simulate NACA four digits equations to get the coordinates for the whole series using the mathematical analysis and numerical methods in programing using Matlab. This lead to choose the optimal dimension to avoid curvature discontinuities and getting rid of undefined cases, which leads to get the optimal performance.

2. Research objective

The goal of this research is twofold:

1. Simulating NACA four digits equations that coordinates blade curvature can be obtained.
2. Define new conditions for blade parameters, which lead to avoid discontinuities, undefined cases, and keep the blade smooth.

3. Materials and methods.

3.1 Simulating NACA Equations:

The equation of the camber line is split into two sections the first one at the right side of the point of maximum camber position (P) the second one at the left side of it. In order to calculate the position of the final airfoil skin later the gradient of the camber line is also required.[2]

The coordinates system is:

Z : axis along the blade length

x : coordinates along airfoil length, from 0 to c (which stands for chord length)

y : coordinates above and below the line extending along the airfoil length, where y_t is thickness coordinates and y_c is camber coordinates.

t : maximum airfoil thickness in tenths of the chord (i.e. a 15% thick airfoil would be 0.15).

m : maximum camber in tenths of the chord.

p : position of the maximum camber along the chord in tenths of the chord.

Since blade equations can be considered as surface equations. In a space, two parameters are required to describe these equations, the parameter u varies along the chord line is chosen (instead of x) and the parameter v varies along the blade length is chosen (instead of z) as shown in figure (2).

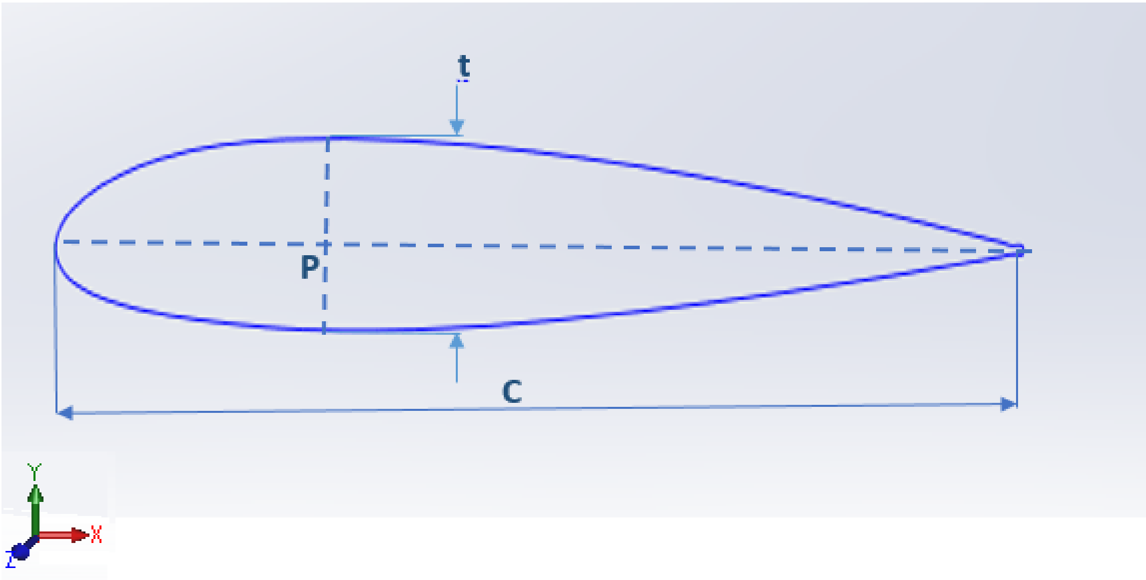


Figure (2) Blade parameters explanation

The flowchart in figure (3) illustrates the used algorithm in the research methodology.

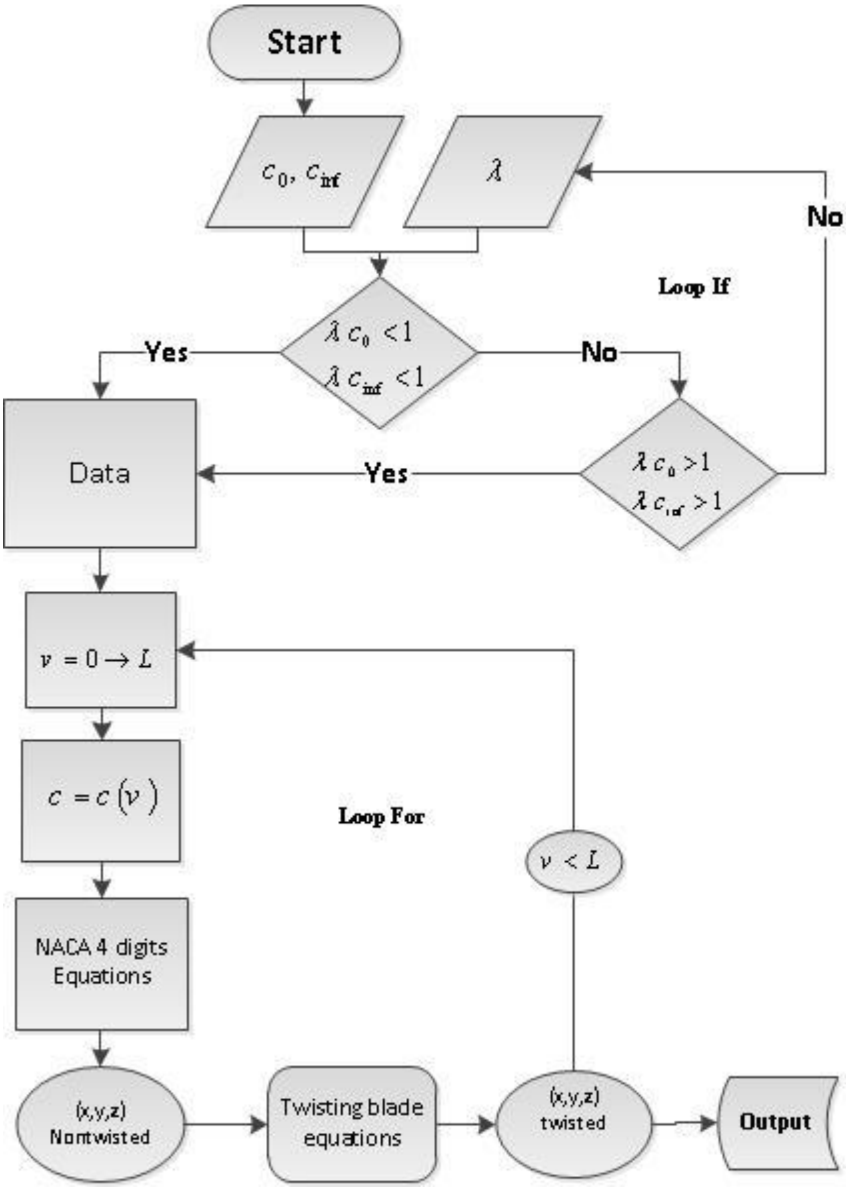


Figure (3) The used algorithm.

The chord line length varies according to the blade length, which is C_0 at the beginning of the blade and $C_{\inf}$ at the end of the blade as shown in figure (4). In order to reduce the blade weight and the centrifugal forces caused by rotation, it is adopted a linear variation of the chord (c) as a function to blade length, and the value of the chord will be a linear function with the blade length:

$$c = c_0 - (c_0 - c_{\inf})\frac{v}{L_b}, \quad (1)$$

Where, L_b is the blade length.

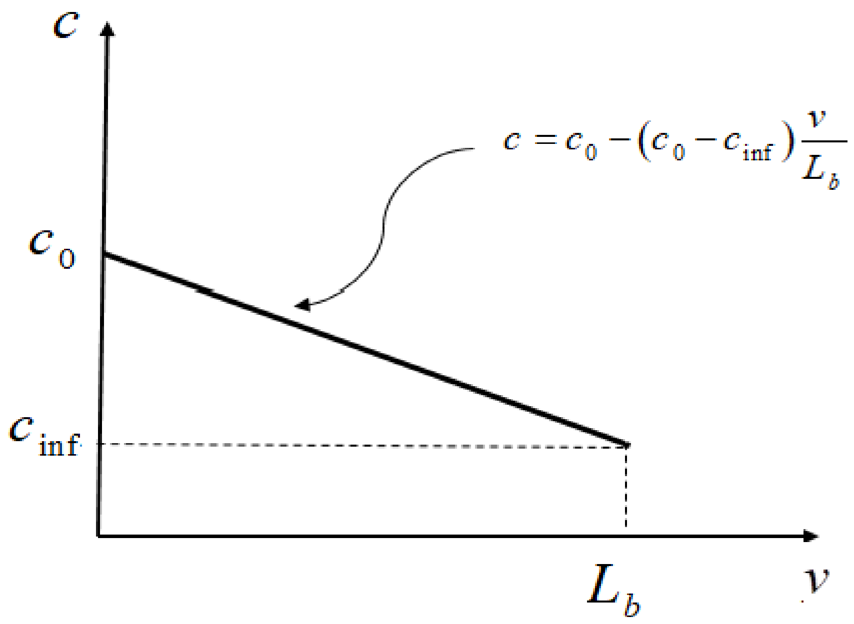


Figure (4) Chord length variation according to blade length.

After determining the value of the chord c, the parameters (t, m, p) can be calculated from the data shown in NACA's explanation [2],

$$t = 0.12c\,, \qquad (2)$$

$$m = \mu c\,, \qquad (3)$$

$$p = \lambda c\,, \qquad (4)$$

Where, $\mu = 0.02$ is the maximum camber in the tenths of the chord, and λ is a parameter determines the location of the maximum thickness of blade section along the chord line and it is mainly related to profile curvature and very important in smoothness and continuation of blade curve.

The thickness distribution above (+) and below (-) the mean line can be calculated by substituting the value of t into the following equation for each of $x \equiv u$ coordinate:

$$y_t = t\, a^t\, q\,, \qquad (5)$$

(a, q) Are two auxiliary vectors given by,

$$a = \langle +0.2969 \quad -0.1260 \quad -0.3516 \quad +0.2843 \quad -0.1015 \rangle^t\,, \qquad (6)$$

$$q = \sqrt{\frac{u}{c}} \quad -\left(\frac{u}{c}\right) \quad -\left(\frac{u}{c}\right)^2 \quad +\left(\frac{u}{c}\right)^3 \quad -\left(\frac{u}{c}\right)^{4t}\,, \qquad (7)$$

The final coordinates of the upper airfoil surface (x_U, y_U), and lower surface (x_L, y_L) can be defined using the following equations, for upper surface:

$$x_U = u - y_t \sin(\theta)\,, \qquad (8)$$

$$y_U = y_c + y_t \cos(\theta)\,, \qquad (9)$$

In addition, for lower surface:

$$x_L = u + y_t \sin\theta, \quad (10)$$

$$y_L = y_c - y_t \cos\theta, \quad (11)$$

Figure (5) shows the coordinates of upper and lower surface of blade curvature.

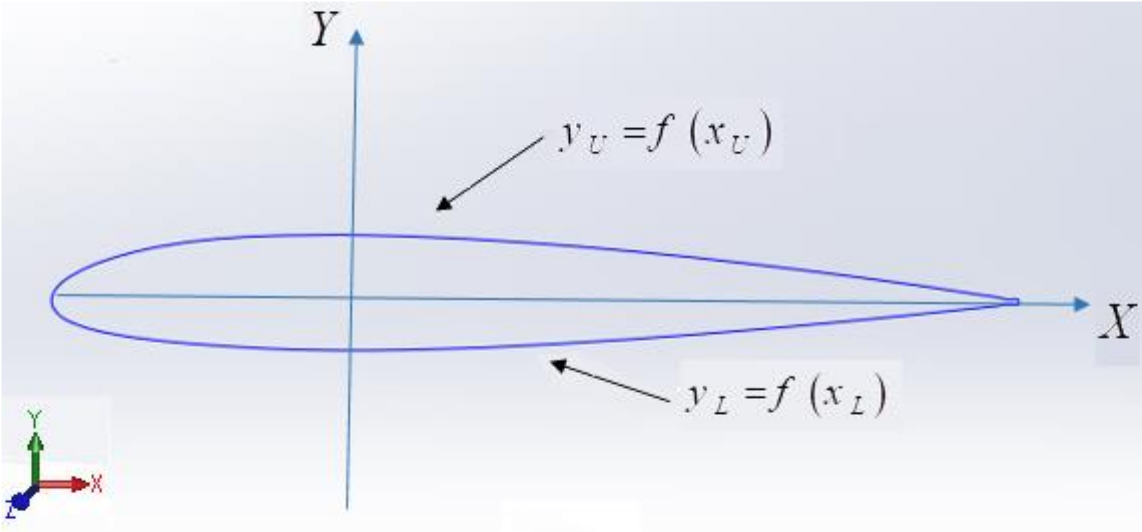


Figure (5) Upper and lower curvature coordinates.

Where, y_c is the mean camber line coordinate, it can be calculated from the following equation:

$$y_c = y_{c1}\boldsymbol{\sigma}(cp - u) + y_{c2}\boldsymbol{\sigma}(u - cp), \quad (12)$$

Where, $\boldsymbol{\sigma}$ is Heaviside function, which is useful in maintaining the continuity of the function which is composed of two curves (y_{c1}, y_{c2}) at the connection node. I.e. $\sigma(u - u_0) = 1$ whatever $u > u_0$ and $\sigma(u - u_0) = 0$ whatever $u < u_0$ where u_0 is the abscissa of connection node.

(y_{c1}, y_{c2}) can be calculated by substituting the values of *m* and *p* from equations (3, 4) in the following equations for each value of *u* corresponds to its x coordinate,

$$y_{c1} = \frac{c\,m}{p^2}\left(2p\frac{u}{c} - \left(\frac{u}{c}\right)^2\right), \quad (13)$$

$$y_{c2} = \frac{c\,m}{(1-p)^2}\left((1-2p) + 2p\frac{u}{c} - \left(\frac{u}{c}\right)^2\right), \quad (14)$$

Figure (6) shows blade sections illustrate camber as a function to the chord line before rotating the coordinates obtained from simulation using numerical algorithm with m- file technique in Matlab as shown in the flowchart above.

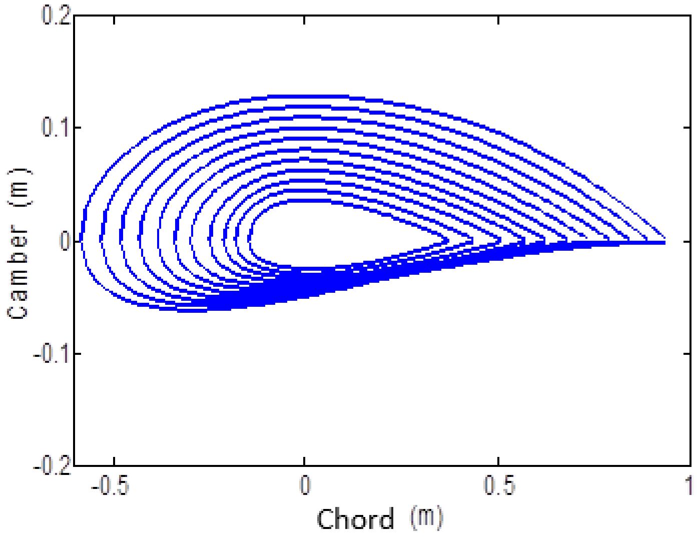


Figure (6) Blade sections

Finally, Heaviside function $\boldsymbol{\sigma}(cp-u)$ is used in the syntax because the mean camber line y_c is not a smooth function, (it is composed of two different curves (y_{c1}, y_{c2})), so it takes the value y_{c1} in the first interval where $(u<cp)$, the value y_{c2} in the second interval where $(u>cp)$, and the angle θ in equations (8, 9, 10, 11) can be calculated from NACA formula,

$$\theta=\arctan\left(\frac{dy_c}{du}\right), \qquad (15)$$

By deriving equation (12) as a function to u, we find

$$\frac{dy_c}{du}=\frac{dy_{c1}}{du}\boldsymbol{\sigma}(cp-u)-y_{c1}\boldsymbol{\delta}(cp-u)+\frac{dy_{c2}}{du}\boldsymbol{\sigma}(u-cp)+y_{c2}\boldsymbol{\delta}(u-cp), \qquad (16)$$

Where $\boldsymbol{\delta}$ is Dirac function (it is a result from derivation of the Heaviside function) defined by,

$$\boldsymbol{\delta}(u)=\frac{d\boldsymbol{\sigma}(u)}{du}, \qquad (17)$$

As a result, the airfoil curvature can be drawn using successive sections of the blade profile figure (6) along the blade length before rotation as shown in figure (7).

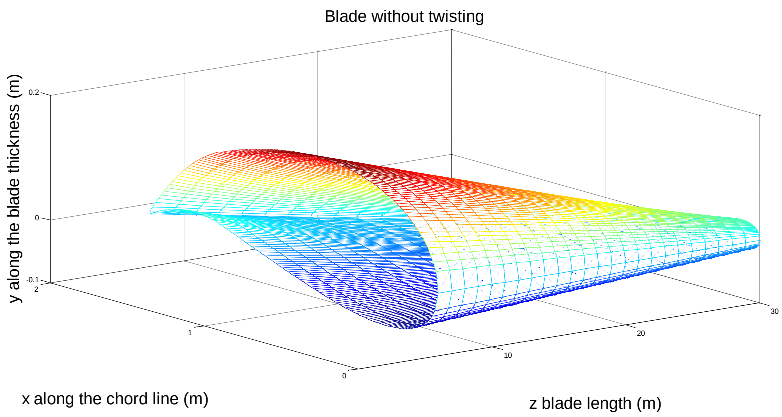


Figure (7) Blade before twisting.

3.2 Twisting blade surface

Likewise, the twist angle of the blade changes along the length of blade:

$$\alpha = \beta - \alpha_0 \left(\frac{v}{L_b} \right), \qquad (18)$$

Where β is angle of attack.

Similarly, equation (18) is like equation (1) where the angle (α) changes linearly along blade length.

After calculating the coordinates of the non- twisted blade from the equations (8, 9, 10, and 11) and calculating the twisting angle for all of sections from equation (18), we can find the coordinates of the twisted blade using the rotation relationships in the plane [2] as the following:

$$\begin{pmatrix} x_{Tws} \\ y_{Tws} \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} x_{N-Tws} \\ y_{N-Tws} \end{pmatrix}, \qquad (19)$$

A rotation matrix is a transformation matrix that is used to perform a rotation between two coordinates systems (x_{Tws}, y_{Tws}) and (x_{N-Tws}, y_{N-Tws}), in which the profile sections can be twisted along the blade length. Figure (8) shows blade profile after rotating sections.

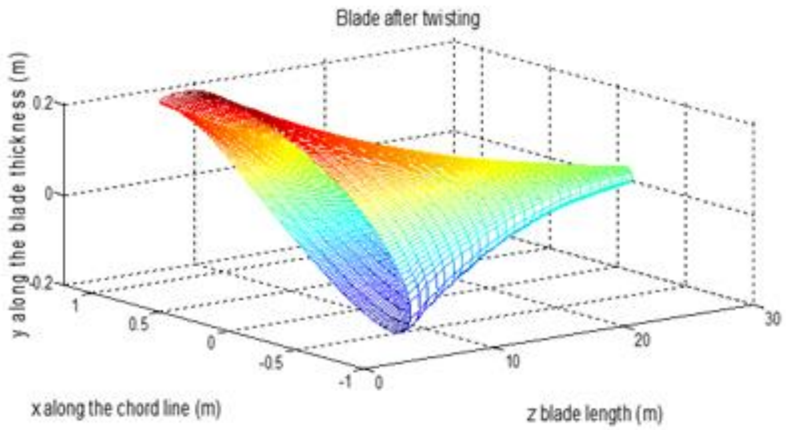


Figure (8) Blade after twisting.

3.3 Conditions of blade surface smoothness

In order to reduce the weight of the blade and the aerodynamic forces, the chord line value c varies according to the length of the blade see equation (1).

Therefore, by substituting (1) into the two relationships (3, 4) m and p become as the following:

$$m = \mu \left(c_0 - (c_0 - c_{\inf}) \frac{v}{L_b} \right) \qquad (20)$$

$$p = \lambda \left(c_0 - (c_0 - c_{\inf}) \frac{v}{L_b} \right) \qquad (21)$$

And according to the relationships (12, 13, 14) (y_c) becomes,

$$y_c = y_{c1}(u,v)\, \boldsymbol{\sigma}_1 (\lambda c^2 - u) + y_{c2}(u,v)\, \boldsymbol{\sigma}_2 (u - \lambda c^2), \qquad (22)$$

The angle (θ) can be calculated from equation (15),

$$\theta = \arctan\left(\frac{\partial y_{c1}}{\partial u}\boldsymbol{\sigma}_1\left(\lambda c^2 - u\right) - y_{c1}\boldsymbol{\delta}_1\left(\lambda c^2 - u\right) + \frac{\partial y_{c2}}{\partial u}\boldsymbol{\sigma}_2\left(u - \lambda c^2\right) + y_{c2}\boldsymbol{\delta}_2\left(u - \lambda c^2\right)\right), \quad (23)$$

Where the variable of Heaviside function (ξ) given as:

$$\xi_1 = \lambda c^2 - u, \quad (24)$$

for the first Heaviside function where ($u < c\ p$), and

$$\xi_2 = u - \lambda c^2, \quad (25)$$

for the second one where ($u > c\ p$).

Since the Heaviside function is a non- smooth function. Therefore, the variable of each Heaviside function (ξ) must maintain on its sign along the blade length, which keeps it in its region. As (ξ) is a function of two variables (u, v), we will accept some kind of approximation in this issue.

Since the maximum value of (u) is equal (c), so the variable of Heaviside function becomes,

$$\xi_1\big|_{u=\max} = \lambda c^2 - c, \quad (26)$$

In order to avoid the point in which (ξ) changes its sign, we have to determine this point. To this end, we should substitute (c) from equation (1) in equation (26) and make (ξ) is equal to zero,

$$\lambda\left(c_0 - \left(c_0 - c_{\inf}\right)\frac{v}{L_b}\right) - 1 = 0, \quad (27)$$

At the beginning and end of the blade for (ξ_1) we find,

$$\begin{aligned} v &= 0 \rightarrow \lambda\, c_0 - 1 = 0 \\ v &= L \rightarrow \lambda\, c_{\inf} - 1 = 0 \end{aligned}, \quad (28)$$

In the same manner for (ξ_2) we find,

$$\begin{aligned} v &= 0 \rightarrow 1 - \lambda\, c_0 = 0 \\ v &= L \rightarrow 1 - \lambda\, c_{\inf} = 0 \end{aligned}, \quad (29)$$

4. Results.

As a result, from equations (28, 29) we find that to avoid the undefined cases, which lead to discontinuities in blade curvature, conditions of smoothness, and continuity of the surface would be given as the following:

$$\lambda\, c_0 < 1 \quad \& \quad \lambda\, c_{\inf} < 1, \quad (30)$$

Or

$$\lambda\, c_0 > 1 \quad \& \quad \lambda\, c_{\inf} > 1, \quad (31)$$

In fact, these conditions (30, 31) are equivalent to hyperbola equation in the coordinate system associated of its two asymptotes (λ, c). Therefore, the locus of the points of this hyperbola should not be used to avoid undefined cases and discontinuity problems.

Consequently, it is very effective to choose the values of chord line at the blade root and its end, and the position at the chord line corresponds to maximum thickness from the ranges defined above, in avoiding geometry problems (i.e. discontinuities).

Figure (9) shows two blades the first one is with continuous curvature where equations (30, 31) are valid, and the second one is with discontinuous curvature.

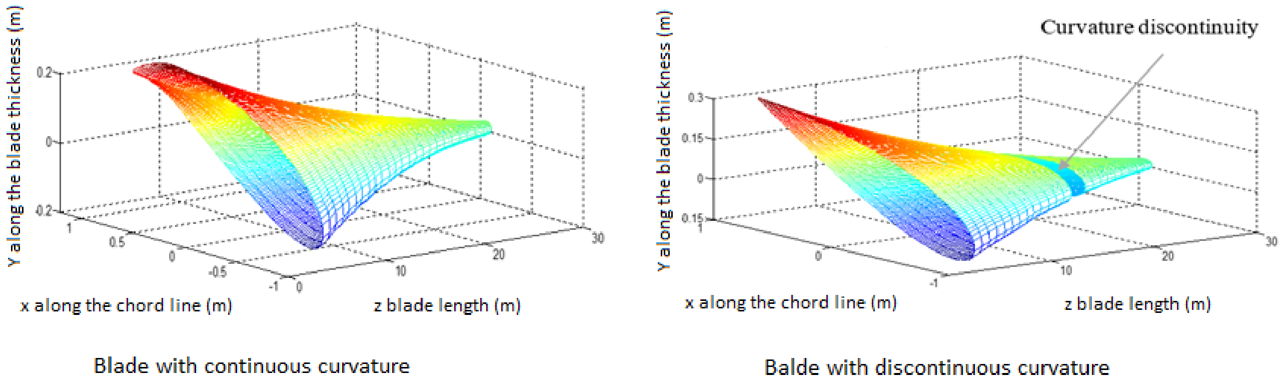


Figure (9) Blade with two different cases.

5. Discussion.

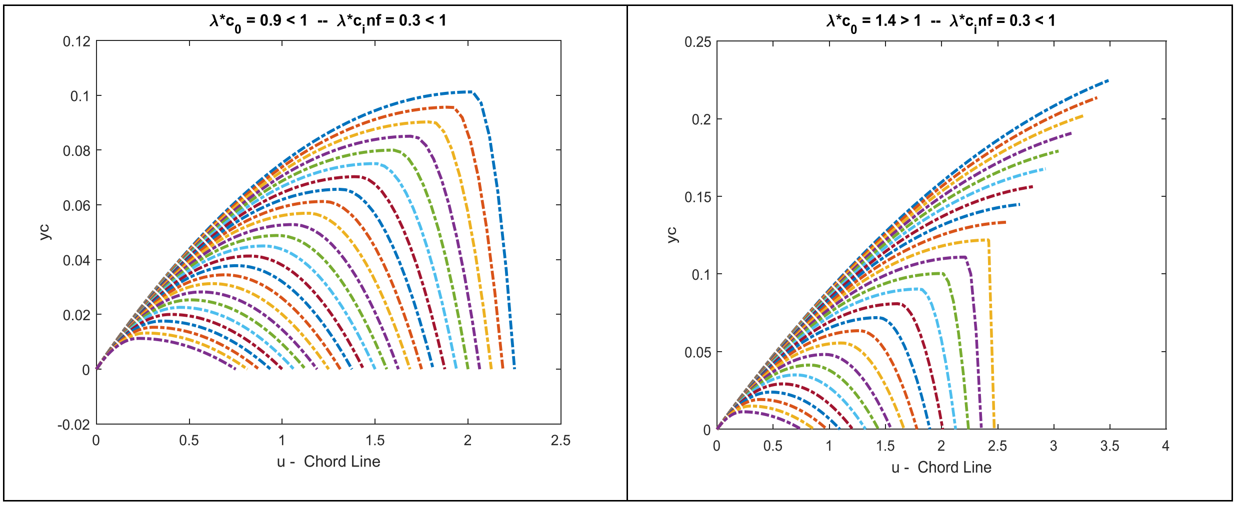
Fig(A) in Figure(10) shows a case where c_0, c_{inf}, λ are chosen that the conditions of smoothness and continuation in curvature are satisfied and valid as shown in equation (30),

$$\lambda\, c_0 = 0.9 < 1 \quad \& \quad \lambda\, c_{\inf} = 0.3 < 1$$

Fig(B) in Figure(10) shows a case where c_0, c_{inf}, λ are chosen out of the range, which we have determined in equation (30,31),so that the conditions of smoothness and continuation in curvature are not valid,

$$\lambda\, c_0 = 1.4 > 1 \quad \& \quad \lambda\, c_{\inf} = 0.3 < 1$$

It is obvious that the obtained conditions in equations (30, 31) are simple, not complicated and have effective influence of blade geometry, which help to choose the suitable values of blade parameters. The selection of blade parameters within the ranges that we have concluded in our research enables us to get refined blade, which does not suffer any discontinuity in the geometry or undefined case, and consequently, it helps in facilitating the writing of the necessary programs in MATLAB or any other software used in simulation.



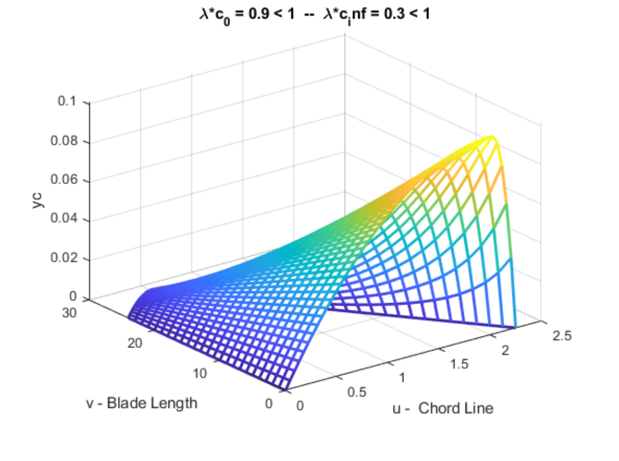


Fig- A

Conditions of continuation are satisfied

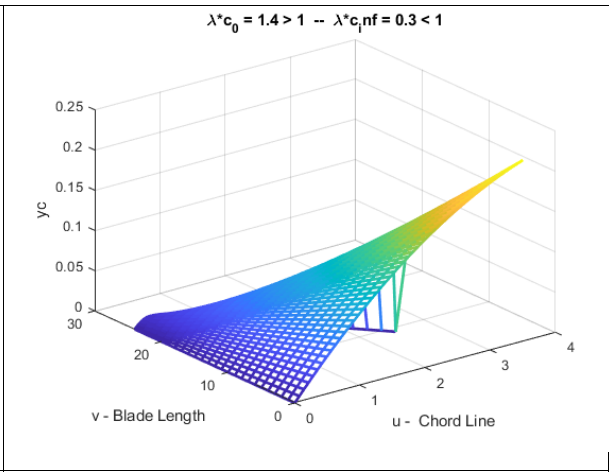


Fig- B

Conditions of continuation are not satisfied

Figure (10) Conditions of continuation.

6. Conclusion.

Simulating an airfoil blade (NACA Equations Method) using MATLAB helps us to change any parameter in the blade (twist angle, angle of attack, the value of the chord line at the beginning and at the end of the blade, blade length,....etc.) and get any blade coordinates directly for the whole series of NACA four digits.

The appropriate selection of the blade parameters according to the conditions concluded in our research makes the blade surface smooth, which leads to obtain continuous flow on the blade surface and thus prevents separation of the boundary layer. This helps to reduce energy losses, and get high efficiency in the wind turbine.

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