Journal of Engineering Sciences
& Information Technology
Issue (1), Volume (2)
March 2018
ISSN: 2522-3321

المجلة العربية للعلوم و نشر الأبحاث
Arab Journal of Sciences & Research Publishing
AJSRP

An Analytical Approach for Electronic Properties of Semiconductors Quantum Well Wires

Ahmad Salim Hussain Nassar

Palestine Technical University || Khadoorie (PTUK) || Ramallah Branch || PALESTINE

Abstract: In this work a theoretical investigation of the electronic properties of quasi-one dimensional semiconductor systems is presented. An approximately analytical self-consistent approach is used for the calculation of the energy subbands and electron density. The electron – impurity and electron – electron interaction potentials are calculated for a two subband model, for a cylindrical quantum wire. The analytical results for the electron –impurity and electron-electron interaction potentials are in good agreement with the results of the model calculations. With my analytical results I discuss various aspects of the electronic properties of semiconductors quantum well wires, such as the binding energy of shallow impurity, screening, intrasubband Plasmon, and the mobility.

Keywords : Electronic properties, Shallow impurity, Semiconductors, Quantum well wires, One dimensional systems .

I. Introduction

Quantum well structures in which electrons have freedom of motion in two, one, and zero directions, are of great current interest as they promise faster, smaller and lower-power consuming devices(Weisbuch, C., Vinter, B., & Brennan, K. F., 1992). Devices based on structures permitting two-dimensional (2D) motion are now fairly well developed and are even commercially available (Harrison, P. & Valavanis, A. 2016). Effort is also being made to realise devices using one-dimensional (1D) motion, which promise better properties. (Haug, H. & Koch, S. W. 2009). Fabrication of the so-called one-dimensional (1D) or quantum wire structures have been reported (Gershoni, D. et al., 1988;Notomi, M. et al., 1991; PTsuchiya, M. et al., 1989). Electronic properties of quasi one-dimensional systems (Q1DS's) have been recently studied experimentally (Kallin, C., Leavens, C. & Taylor, R. 1988).Usually, quasi one-dimensional structures are produced by an additional confinement of a two-dimensional electron gas as realized in silicon metal-oxide semiconductor structures or in $Al_xGa_{1-x}As/GaAs$ heterostructures. For realistic calculations of the subband structure in these systems, one must solve a two-dimensional Schrodinger equation and a two-dimensional Poisson equation, and only numerical results are available (Laux, S.E., Frank, D.F& Stern, F., 1988). Analytical structure of Q1DS's can be obtained for a cylinder of radius R_o with infinite barriers(Nag, B. & Gangopadhyay, S., 1993). However, the barrier potential is finite when the wire is surrounded by another semiconductor(Notomi, M. et al., 1991). An analytical solution of the equation for the envelope function is not possible for such wires with Cartesian geometry. When the

barrier potential is infinite as in structures with free surfaces, analytical results for the energy subbands for cylindrical and rectangular wires can be obtained(Gold, A. & Ghazali, A., 1990). Even for these models the Fourier transform of the electron-electron interaction potential which plays a fundamental role for Plasmon, has mainly been given as numerical result (Kodama, T.& Osaka, Y., 1986).For the lowest subband and for a cylindrical wire, analytical results for the electron-impurity and electron-electron interaction potentials have been obtained with the approximation of constant electron density in the wire(Fishman, G., 1986).

In this paper I present analytical and numerical results for the energy subbands and electron density by an approximately self consistent calculation. Analytical and numerical results for electron – impurity and electron-electron interaction potentials for two lowest subbands of a cylindrical wire is also presented .The good match of those analytical results with the numerical calculation has motivated us to study various electronic properties using the analytical results. I discuss the binding energy of shallow impurity states, the elementary calculation within the variational approach (Nag, B., & Gangopadhyay, S. 1993). For this calculation I use the separable-potential approximation (Gold, A. & Ghazali, A. 1990). Mobility limits for charged-impurity scattering in Q1DES's have been calculated by (Fishman, G.,1986) within some more sophisticated models. These calculations will be discussed according to my analytical result for electron-impurity interaction potential.

PROBLEM STATEMENT:

Quantum structures, nanoscale layers, channels and boxes known as quantum wells, quantum wires and quantum dots turn the board energy bands of conventional semiconductors into more sharply defined energy levels(Harrison, P. 2016). That is a transformation which promises greater speed and efficiency for resulting circuits and optical devices (Burileanu, LM& Radu, A. 2011)..

The basic principle behind quantum wells, wires and dots, is the same. Confine electrons in a restricted region of semiconductors by hemming it with another semiconductor that has a higher bandgap,a measure of the amount of energy that has to be pumped into the material to get electrons flowing. Electrons will naturally tend to flow in the confined region, where the bandgap is lower. In quantum wells, that region is often a 100Å to 200Å thick slice of semiconductor GaAs, built up by vapor deposition on a base of higher bandgap material AlGaAs. A second layer of AlGaAs closes the top. Confined in the slice, the electrons so have little space to move that their energy states are forced to cluster around specific peak (X Zhao, CM Wei, L Yang, MY Chou – 2004).

Right now the fabrication of Quantum Well Wires look more promising as vehicles for studying the physics of confined electrons than as the basis of future technologies because they are complex and expensive to make .

THE OBJECTIVES :

1- To introduce analytical expressions to use by researchers will certainly simplify the study of electronic properties of QWWs .

2- To focus the attention of researchers that QWWs are important and form the basis of future technologies.

METHODOLOGY

Usually, quasi-one dimensional structures are produced by an additional one- dimensional confinement of a two- dimensional electron gas as realized in silicon metal oxide – semiconductor structures or in $AL_XGa_{1-x}As/GaAs$ heterostructures. (Aciksoz, E, Bayrak,O Soylu,A - Chinese Physics B, 2016).

In this work, I am interested in calculating the electronic structure of quantum well wires. For realistic calculations of the subband structure in these systems, one must solve a two – dimensional Schrodinger equation and a two dimensional Poisson equation, and mainly numerical results are available(Nag, B., & Gangopadhyay, S. 1993). Analytical results for the subband structure of quasi one – dimensional electron systems can be obtained for a cylinder of radius R_o with infinite barrier and for a rectangular wire with sides L_1 and L_2 with infinite barriers. Similar calculation can be done with finite barriers.

For my calculation of the subband structure in a cylindrical wire of radius R_o, I solve two dimensional Poisson equation and two dimensional Schrodinger equation in cylindrical coordinates in a self consistent manner. By solving Poisson's equation I obtain the self consistent potential to be substituted again into Schrodinger equation . The solution of the Schrodinger equation gives, the energy subbands as a function of the wire radius R_o and the electron density N for infinite and finite potential barrier models. The solution of Schrodinger equation without including the self consistent potential is a nonlinear equation in(E). I obtain also the wave function in terms of Bessel's function. The analytical self consistent solution for the energy subband is again a function of R_o and N as will be shown later(Nag, B., & Gangopadhyay, S. 1993).

In the following sections the electronic properties of quasi one dimensional electron systems are studied by an approximately analytical self consistent solution of the Schrodinger and Poisson's equation. The binding energy for shallow impurity is calculated within the separable potential approximation (SPA). Analytical expression for electron –impurity interaction potential is also derived. Plasmons (Nozières, P., & Pines, D. 1999) . Mobility limits for charged impurity in quasi one dimensional electron systems have been discussed according to my analytical results for electron –impurity and electron-electron potentials. Finally I compare the results of infinite and finite potential barrier models.

II. DISCUSSION

III. ENERGY SUBBANDS:

An approximately analytical self consistent calculation of the subband energies is presented by solving two dimensional Schrodinger equation and two dimensional Poisson equation. In my model the quasi one –dimensional structure is taken as a circular cylinder of radius R_o. Consider a finite barrier potential V(r) at r =R_o, for r <R_o (V_o= 0) and (V_o)for r>Ro .The motion of the electrons is restricted in the (r,θ) plane, while the motion in the z-direction is free. A constant electron mass is assumed, the boundary conditions on the wave function are that Ψ and its first derivative are continuous at the boundary. The solution of the corresponding Schrodinger equation expressed in term of ordinary J_l (x) and modified Bessel function K_l (x) (Gradshteyn,I.S.&Ryzhik, I.M.,1980).

$$\psi_{nlk}(r,\theta,z)=\begin{cases} N_c e^{ikz} e^{il\theta} J_l(r_{nl},r), r \le R_o, \\ \dfrac{N_c J_l(r_{nl}R_o) e^{ikz} e^{il\theta}}{K_l(b_{nl}R_o)} K_l(b_{nl},r), r \ge R_o \end{cases}, \qquad (1)$$

Where N_c is the normalization constant,

$$r_{nl} = \sqrt{\frac{2m^* E_{nl}}{h^2}}, \qquad (2)$$

and

$$b_{nl} = \sqrt{\frac{2m^*(V_o - E_{nl})}{h^2}}, \qquad (3)$$

For the first subband, after applying the boundary conditions we get,

$$\frac{b_{10}}{r_{10}} = \frac{K_o(b_{10}R_o)J_1(r_{10}R_o)}{J_o(r_{10}R_o)K_1(b_{10}R_o)} \qquad (4)$$

Equation (4) is a non-linear equation, only numerical solution is available, with the parameters $R_o = 5a^*, V_o = 57.4Ry^*, E_{10}$ is found to be equal to (3Ry*).The filling of the subbands introduces an additional potential which can be approximated as a Hartree term. If we assume that only the lowest subband occupied by electrons, the finite electron density gives rise to a potential Φ(r), which can be calculated by the Poisson equation:

$$\frac{d^2}{dr^2}\Phi(r) + \frac{1}{r}\frac{d}{dr}\Phi(r) = \frac{-4\pi e^2}{\varepsilon_L} N|\psi|^2 \qquad (5)$$

where N is the one-dimensional electron density.

From Eqs.(1) and (5) the Hartree potential can be found, then for r <R_o and r >R_o.

The correction of the energy due to this potential is

$$\Delta E_1 = -1.84 N_1, \qquad (6)$$

where N is the electron density in the first subband.

The first subband is,

$E_1 = 3\text{-}1.84N_1$ **(7)**

For the second subband by using Eq. (1) and the boundary conditions, we get,

$$\frac{\beta_2}{\alpha_2} = \frac{\left[\frac{J_1(\alpha_2 R_o)}{\alpha_2 R_o} - J_o(\alpha_2 R_o)\right] K_1(\beta_2 R_o)}{\left[K_o(\beta_2 R_o) + \frac{K_1(\beta_2 R_o)}{\beta_2 R_o}\right] J_1(\alpha_2 R_o)}, \quad \textbf{(8)}$$

Where $\alpha_2 = r_{12}$ and $\beta_2 = b_{21}$.

Equation (8) is solved numerically, E_{21} is found to be equal (6Ry*).

The Hartree potential for r <Ro and r>Ro is calculated from Eq. (5). The correction to the energy due to Hartree term is given by, $\Delta E_2 = -2.32N_1 - 2.57N_2$ **(9)**

where N_2 is the electron density in the second subband.

$E_2 = 6 - 2.32N_1 - 2.57N_2$. **(10)**

The total electron density in the wire is,

$N = N_1 + N_2$ **(11)**

The electron density in each subband is determined from Fermi Dirac distribution function and density of states for one-dimensional systems. It is given by,

$E_F = N_1^2\pi^2 + E_1$, and **(12)**

$E_F = N_{21}^2\pi^2 + E_{21}$ **(13)**

$N_1^2\pi^2 + E_1 = N_{21}^2\pi^2 + E_{21}$ **(14)**

Where E_F is Fermi energy.

Equations (7), (10), (11) and (14) is solved with the parameter N=(1.24)(1⁄a*) we get, N_1= 0.66(-1⁄a*), N_2=0.58(1⁄a*),E_1=1.78 Ry* and EF=6.07 Ry*. The confining potential and energy subbands are shown in Figure (1). The researcher noticed that the finite confining potential reduces the energy subbands.

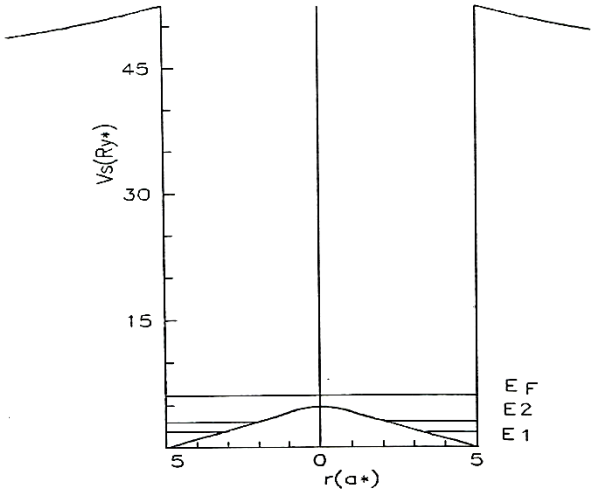

Figure 1: The confining potential and the energy subbands for finite barrier potential

IV.SHALLOW IMPURITIES:

Various electronic properties of the quasi one-dimensional structure are determined by the electron-impurity and the electron-electron potentials. I assume that charged impurities are randomly distributed on the surface of a cylinder with radius R_o. The interaction between point charges follows the Coulomb law. I introduce the Fourier transform for the z-direction with q as a one-dimensional wave vector. For a system with cylindrical symmetry, the interaction potential between an electron at r and an electron at r' is given by,

$$V(r, r', q) = \frac{2e^2}{\varepsilon_L} K_o\left(q|r - r'|\right) \qquad (15)$$

Where K_o is the modified Bessel function of order zero. The effective interactions are weighted with the wave functions. The electron-impurity interaction potential for an impurity located at R=(R,θ) is given by(Gold, A. and Ghazali, A., 1990).

$$V_{ij}(q, R, \theta) = -\int d^2 r' \psi_i^*(r') \psi_j(r') V(R, r', q) \qquad (16)$$

With Eqs. (2) and (11) the electron-impurity interaction potential can be obtained in analytical form,

$$V_{11}(q, R) = \frac{2e^2}{\varepsilon_L} \pi N_c^2 R_o^2 I_o(qR) \left[\ln(qR_o) \left[J_o^2(\alpha R_o) + J_1^2(\alpha R_o) \right] - \left[J_o^2(\alpha R_o) + J_1^2(\alpha R_o) - \frac{J_o(\alpha R_o) J_1(\alpha R_o)}{\alpha R_o} \right] \right],$$

$R<R_o$, **(17)**

Where $\alpha = r_{10}$, R_o is the radius of the wire, R is the position of the impurity, J_o is the Bessel function of order zero and I_o is the modified Bessel function of order zero. The binding energy of shallow impurities in a cylindrical quantum well wire is studied as a function of the impurity location with respect to the axis of the wire. The binding energy variation with respect to the electron density is also calculated. For these calculations I use the separable potential approximation (SPA). In the (SPA) the electron-impurity interaction potential is written as,

$$V_{ii}^{e-i}(q - q') = C(d) \left[V_{ii}^{e-i}(q) \right]^{1/2} \left[V_{ii}^{e-i}(q') \right]^{1/2}, \qquad (18)$$

Where C(d) is a numerical coefficient that depends on the dimensionality of the system. The binding energy (EB) for shallow impurities for d=1 is given by the solution of the following equation,

$$\frac{C(1)}{2\pi} \int_{-\infty}^{\infty} \frac{V_{ii}^{e-i}(q)}{\varepsilon(q) \left[\frac{q^2}{2m* + E_B} \right]} = -1 \qquad (19)$$

where ε(q) is the static dielectric function. For screened impurity case, $\varepsilon(\mathbf{q}) = \varepsilon_{111}(\mathbf{q})$for the first subband which is derived from the dielectric tensor of a multi subband system (Ando, A. & Fowler, F., 1989).

$$\varepsilon_{ijmn}(q, \Omega) = \delta_{ij} \delta_{mn} + X_{ij}(q, \Omega) V_{ijmn}^{e-e}(q) \qquad (20)$$

Where X_{ij} is the generalized Lindhard function for wave vector **q** and frequency Ω.

(in this case Ω=0), δ_{ij} the Kronecker delta function.

$$X_{11}(q,\Omega) = \frac{m^*}{\pi q}\left[\ln\left|\frac{\Omega^2-\Omega_+^2}{\Omega^2-\Omega_-^2}\right|\right] + i\pi\theta(\Omega_+ - \Omega)\theta(\Omega - \Omega_-), \qquad (21)$$

$$\text{and } \Omega_\pm = \frac{\left|q^2 \pm 2K_F q\right|}{2m^*}, \qquad (22)$$

where K_F is Fermi wave number, $K_F = \frac{N_o\pi}{2}$, N_o is the electron density in the first subband, $V_{1111}^{e-e}(q)$ is the electron-electron interaction potential in the first subband, it is derived from the relation,

$$V_{ijk\lambda}(q) = \int d^2r \int d^2r' \psi_i^*(r)\psi_j(r)V(r,r',q)\psi_k^*(r')\psi_\lambda(r'). \qquad (23)$$

By using Eqs. (1) and (15) we obtain,

$$V_{1111}^{e-e}(q,R) = \frac{-2e^2}{\varepsilon_L} N_c^4 R_o^4 \left[\left[J_o^2(\alpha R_o) + J_1^2(\alpha R_o)\right]\left[\ln(qR_o)\right]\left[J_o^2(\alpha R_o) + J_1^2(\alpha R_o)\right] - \left[J_o^2(\alpha R_o) - \frac{J_o(\alpha R_o)J_1(\alpha R_o)}{\alpha R_o}\right]\right] \qquad (24)$$

The binding energy of shallow impurities in two dimensional systems is strongly reduced by screening via unbound carrier. This behavior has been confirmed in the (SPA). With increasing electron density, the binding energy decreases and approaches a constant for large electron density.

For QIDS's the screening effect on the binding energy of shallow impurity states has been calculated by (Kodama, T. & Osaka, Y., 1986) and recently by(ciksoz, E., Bayrak, O., Soylu, A, 2016). The binding energy first decreases and then increases again with increasing electron density.

With the parameters R_0=5a* and N_0=0.66(1\a*), The binding energy versus electron density is shown in figure(2),with impurity located at the wire. The binding energy versus impurity position is shown in figure (3). I use this approach to calculate the binding energy because exactly solvable Coulomb model is not available and only numerical calculation for Eq. (19) is possible.(Montes, A., Duque, CA. 1997).

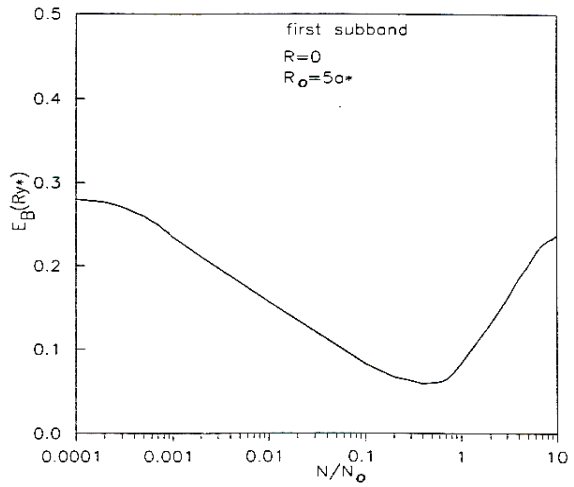

Figure 2: The binding energy versus electron density for finite barrier potential

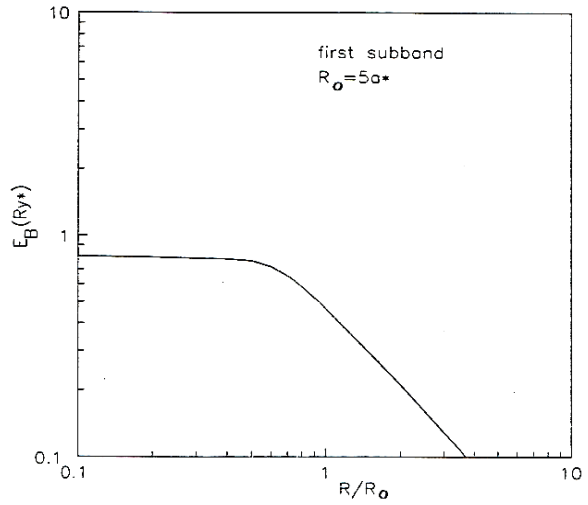

Figure 3: The binding energy versus impurity position for finite barrier potential

V. PLASOMONS

The electron potential describes the screening properties of the wire, as shown in the previous section as an application of the electron-electron potential. I discuss in this section the collective excitation spectrum (Plasmons) for Q1DES´s (.Brataas, A & Mal'shukov, AG 1996). Restriction to a two subband model (i =1,2) and the cylindrical symmetry implies that only four matrix elements of V_{ijkl} (q) are independent and different from zero.

$V_{1111}^{e-e}(q), V_{2222}^{e-e}(q), V_{1221}^{e-e}(q), \text{and} V_{1122}^{e-e}(q).$

Moreover, the following relation hold:

$V_{1122}^{e-e}(q) = V_{2211}^{e-e}(q), V_{1221}^{e-e}(q) = V_{2112}^{e-e}(q) = V_{2121}^{e-e}(q)$.

I shall use Eq.(20) again, since I restrict my discussion to a two subband model, we have two collective modes given by,

$$\left[1 + V_{1111}^{e-e}(q) X_{11}(q,\Omega)\right]\left[1 + V_{1221}^{e-e}(q) X_{12}(q,\Omega)\right] = 0 \qquad \textbf{(25)}$$

A similar model has been discussed for two-dimensional systems(Sarma,S.D,1984). From Eq.(25) there are two collective modes, the first collective mode is the intrasubband Plasmon which is given by (Nozières, P. & Pines, D., 1999, Goni, AR., Pinczuk, A, Weiner,JS Calleja, JM.,Dennis, BS 1991).

$$1 + V_{1111}^{e-e}(q) X_{11}(q,\Omega) = 0 \qquad \textbf{(26)}$$

Intrasubband Plasmon for one dimensional systems have been discussed in(Nozières, P. & Pines, D., 1999). In the following section I present some analytical result for the intrasubband Plasmon.

The real part of $X_{11}(q,\Omega)$ determines the Plasmon dispersion, while the imaginary part determines the particle-hole excitation spectrum. The large frequency expansion of the Lindhard function for Q_1DES´s is expressed as,

$$-X_{11}(q,\Omega)=\frac{Nq^2}{m^*\Omega^2}\left[1+\frac{f_1(q)}{\Omega^2}+0\left[\frac{1}{\Omega^2}\right]\right] \quad , \qquad \textbf{(27)}$$

where $f_1(q) = q^2(q^2+4K_F^2)/4m^*$, **(28)**

In the light of our analytical result Eq.(18) and the use of the following parameters R_o=1a*, N_o=0.57 (1/a*) and N=0.2 N_o. With $V_{1111}^{e-e}(q)$ for qR_o<<1, we get by using Eqs.(27) and(24)

$$\Omega(qR_o<<1)=\left[\frac{2Nq^2V_{1111}^{e-e}(q)}{2m^*}\right]^{1/2} \qquad \textbf{(29)}$$

with $V_{1111}^{e-e}(q)=\frac{e^2}{2\varepsilon_L}f(q)$, we get by using Eqs.(20) and (21),

$$\Omega(qR_o<<1)/\Omega_o=qR_o\left\{\frac{f(q)}{2}\left[1+\left[1+\frac{4f_1(q)}{\Omega_o^2 f(q)(qR_o)^2}\right]^{1/2}\right]\right\}^{1/2} \quad , \qquad \textbf{(30a)}$$

and $\Omega_o^2=\frac{Ne^2}{2\varepsilon_L m^* R_o^2}$ **(30b)**

with f1(q) = 0,

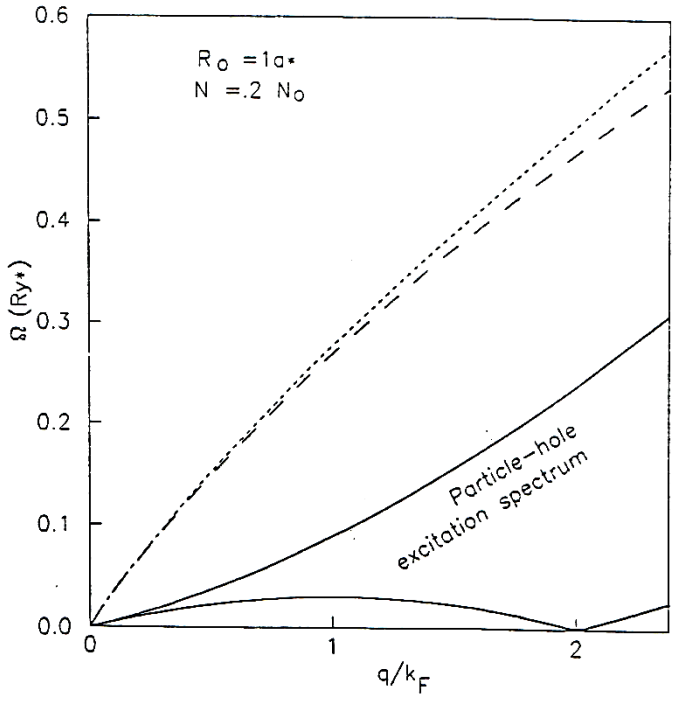

Figure 4: Plasmon spectra for finite potential model .

$$\Omega(qR_o<<1)/\Omega_o=qR_o[f(q)]^{1/2} \qquad \textbf{(31)}$$

In Figure (4) I show the intrasubband energy versus q/K_F(solid line) according to Eq.(26) and the particle hole excitation spectrum versus q/K_F. The dashed line Calculated according to Eq.(29). The dotted line represent Eq.(30a).

In Figure (4) I show the intrasubband energy versus q/K_F (solid line) according to Eq.(26) and the particle hole excitation spectrum versus q/K_F. The dashed line calculated according to Eq.(29). The dotted line represent Eq.(30a).

VI. MOBILITY

A. General Framework

The inverse momentum-relaxation time for zero frequency, zero temperature, and for the lowest subband is given by,(Gold, A. and Gotze, A.,1986)

$$\cdot\frac{1}{\tau(\Omega=0)}=\frac{1}{dm^*N}\sum q^2\frac{\left\langle|U(q)|^2\right\rangle}{[\varepsilon(q)]^2}\phi_{FG}(q,\Omega=0) \qquad (32)$$

here, ‹|U(q)|²› is the average squared Fourier transform of the random potential.

ϕ_{FG} (q,Ω) is the density-density relaxation function of the non-interacting electron gas. (FG) means free gas). And ε(q) is the static dielectric function for the lowest subband. Following the results of (Sarma, S.D., 1984) ϕ_{FG} is given by,

$$\phi_{FG}(q,\Omega=0)=2\pi\rho_F m^* k_F[\delta(2k_F-q)]/q^2 \qquad (33)$$

ρ_F is the density of the states of the free electron gas at the Fermi energy (E_F),

$$\rho_F=\frac{2g_v m^*}{\pi k_F} \qquad (34)$$

From Eqs.(32),(33) and (34) we get,

$$\frac{1}{\tau(0)}=\frac{k_F}{E_F}\frac{\left\langle|U(2k_{F)}|^2\right\rangle}{[\varepsilon(2k_F)]^2} \qquad (35)$$

The mobility in a one-dimensional system can be written as,

$$\mu=\frac{e\tau(0)}{m^*} \qquad (36)$$

For one-dimensional systems, ε(q,T) diverges for $q=2k_F$ and temperature T=0.

According to Eqs.(35) and (36), the mobility would go to infinity. However, at finite Temperature T, ε($q=2k_F$, T>0) is finite and, can be expressed by (Fishman, G., 1986) .

$$\varepsilon(q=2k_{F,T}<<E_F)=1+\frac{g_v}{\pi}\frac{2\varepsilon_L}{e^2}\frac{V_{1111}^{e-e}(2k_F)}{2k_F a^*}[1-G(2k_F)]\ln\left[\frac{2^3e^2E_F}{\pi k_B T}\right], \qquad (37)$$

k_B is Boltzmann constant. I have introduced the local field correction described by G(q=2kF) in Eqs.(5), (6) with the use of Hubbard's approximation. For QIDES's G(q) is expressed as, G(q)=

$$\frac{1}{2g_v}V_{1111}^{e-e}(q)\left[\frac{(q^2+k_F^2)}{V_{1111}^{e-e}(q)}\right]^{1/2} \qquad (38)$$

Eqs.(36)-(37) define the mobility of QIDES's in the lowest order Born approximation for unspecified random potential.

In the following I discuss the mobility limits for remote impurity doping and Homogeneous-background doping.

B. Remote-Impurity Doping

We assume that, impurities are randomly distributed on a cylinder with radius R. The random potential for remote doping (RD) is expressed as,

$$\langle \left| [U(q)]_{RD} \right|^2 \rangle = N_i \left[V_{11}^{e-i}(q) \right]^2 \qquad \textbf{(39)}$$

N_i is the one dimensional impurity density. The mobility is given by,

$$\mu_{RD} = \frac{\pi}{g_v} \frac{e}{h} (a^*)^2 \frac{N}{N_i} \left(\frac{e^2}{2\varepsilon_L} \right)^2 \frac{\left[\varepsilon(q = 2k_F, T) \right]^2}{\left[V_{11}^{e-i}(2k_F) \right]^2} \qquad \textbf{(40)}$$

For GaAs wire with a*=100 A° and g_v=1, the prefactor in Eq.(40) becomes,

$$\pi \frac{e}{h} (a^*)^2 = 4.78 x 10^3 \frac{cm^2}{V.s} \ .$$

The calculated mobility for remote-impurity doping versus electron density is shown in figures .(5),(6) The impurity density is fixed at $N_i = 1x10^6\, cm^{-1}$. With increasing electron density the mobility increases due to the weaker electron-impurity interaction. With increasing (R) the density dependence of the mobility increases due to strong decrease of the electron-impurity potential. Numerical results for mobility of QIDES's for impurity scattering have been presented (Weng,Y. & Leburton,J.P., 1989).

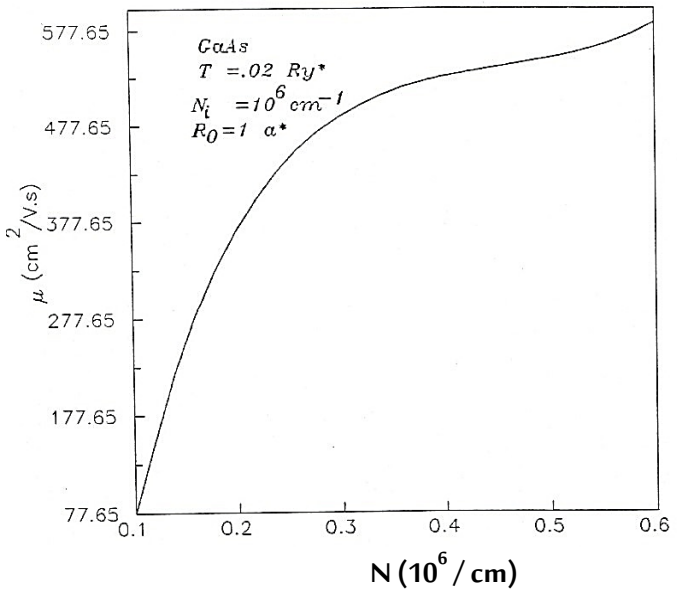

Figure 5: The mobility versus electron density for remote impurity doping (R<R)

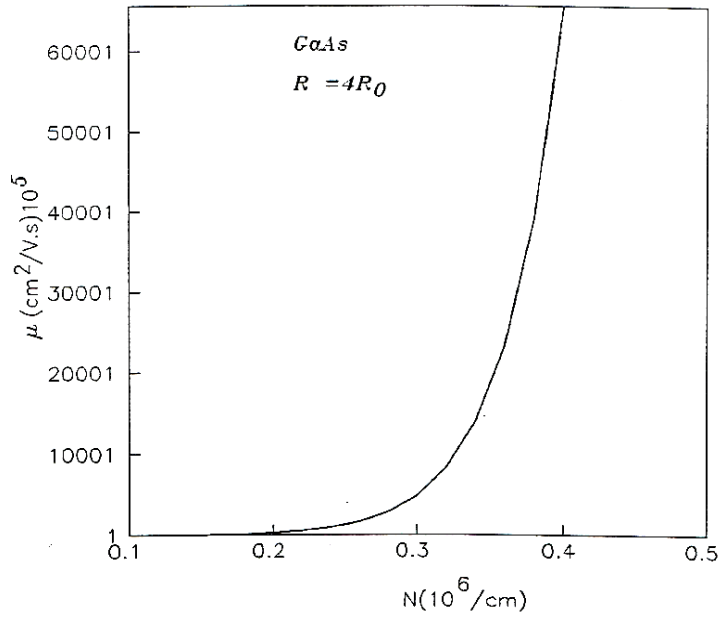

Figure 6: The mobility versus electron density for remote impurity doping(R=4R$_o$)

C. Homogeneous-Background Doping

For homogeneous-Background (B) doping we consider two cases. For cases 1(B_1), impurities are homogeneously distributed in the wire (0<R<R_o). The three dimensional impurity density is N_{B_1}.

For case 2 (B_2) the impurities are homogeneously distributed outside the wire (R>R_o). The three dimensional impurity density is N_{B_2}.

The random potential is defined as,

$$\langle \left| [U(q)]_{B1,B2} \right|^2 \rangle = N_{B1,B2} \int_0^\infty dr \left| V_{11}^{e-i}(q,r) \right|^2 \quad (41)$$

$$V_{11}^{e-i}(q,r) = (1.96 \ln(q,R_o) - 2.19) I_o(q,r) \frac{e^2}{\varepsilon_L} \quad (42)$$

The random potential by using Eqs. (41), (42) is given by,

$$\langle \left| [U(q)]_{B1} \right|^2 \rangle = \left(\frac{e^2}{\varepsilon_L} \right) N_{B1} \frac{R_o^2}{2} \left[(1.96 \ln(qR_o) - 2.19)^2 (I_o^2(qR_o) - I_1^2(qR_o)) \right] = F_{B1} \quad (43)$$

For R>R_o ,

$$\langle \left| [U(q)]_{B2} \right|^2 \rangle = 0.039 N_{B2} \left(\frac{e^2}{\varepsilon_L} \right)^2 \frac{R_o^2}{2} \left(K_1^2(qR_o) - K_o^2(qR_o) \right) = F_{B2} \quad (44)$$

The mobility for homogeneous background doping is expressed as,

$$\mu_{B1,B2} = \frac{\pi}{g_v} \frac{e}{h} (a^*)^2 \frac{N}{N_{B1,B2}} \left(\frac{e^2}{2\varepsilon_L} \right) \frac{[\varepsilon(2k_F,T)]^2}{F_{B1(B2)}(2k_F)} \quad (45)$$

The mobility versus electron density for cases 1 and 2 is shown in Figs.(6), (7), with the parameters, R_o =1a*,N=$N_{B1(B2)}\frac{R_o^2}{2}$,k_BT=0.02 Ry*. For N>1 × 10^5 cm^{-1} .

he result are consistent with the condition T<E_F , in Eq.(37).

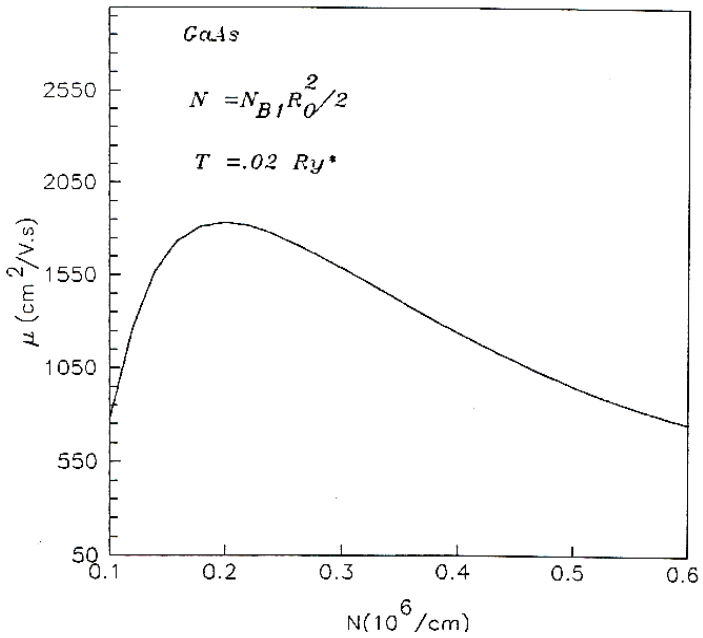

Figure 7: The mobility versus electron density for homogeneous background doping(R<R_o)

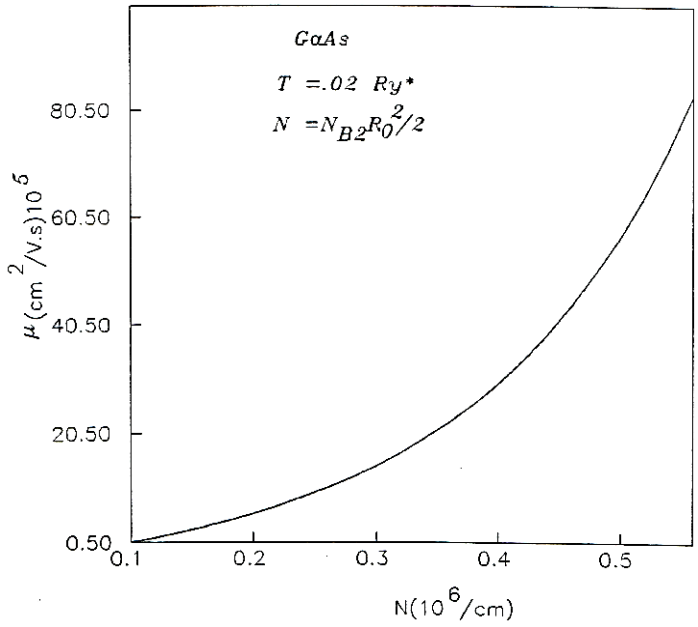

Figure 8: The mobility versus electron density for homogeneous background doping(R>Ro)

CONCLUSION

We have calculated the energy subbands, electron density and Fermi energy for finite potential barrier models. My analytical result is derived within an approximately self consistent approach. The results for finite barrier model are compared with the results of the infinite barrier model. The finite confining potential reduces the subband energies in comparison to the energies in the infinite barrier approximation. I conclude that my model is reasonable and in agreement with numerical calculations. For variational calculations of shallow impurities, wires of circular cross section were chosen to minimize the expected numerical integrations; also it is thought that this shape would best approximate a real wire. However, in this quantum size limit the cross-sectional shape should not significantly affect the wire's characteristics, because the carriers are then confined to move in essentially a single dimension. It is assumed that the wire is sufficiently long and that motion along the wire's axis of symmetry is free; i.e. the confining potential is a function only of a radial co- ordinate. Analytical results for the electron-impurity and electron-electron interaction potentials and for band bending for quasi one dimensional electron systems in a cylindrical semiconductor quantum wire have been presented. I have demonstrated that these analytical results are useful for the calculation of electronic properties.

I have discussed shallow impurities and the effect of screening on the binding energy of shallow impurity. I have seen that the binding energy decreases as the electron density increases till it reaches a minimum value and then increases again, and this result is in very good agreement with (Gold, A. and Ghazali, A., 1990) for the infinite potential model. The binding energy as function of impurity position with respect to the wire axis, and it is in a very good agreement with (Gold, A. and Ghazali, A., 1990).

Plasmon excitation is also discussed for infinite and finite potential models .My result is derived by the use of my analytical form of the electron-electron interaction potential, for finite potential model, it is in agreement with the infinite model case.

But my results are slightly bigger, especially for the Plasmon dispersion. My analytical result for electron –electron interaction potential for finite model differs by a factor of (2) in the logarithmic term compared to the expression of (Gold, A. and Ghazali, A., 1990) for infinite model.

The calculated mobility with the inclusion of Hubbard's local field correction to the random phase approximation is smaller than the mobility calculated by(Friesen, W.I. and Bergersen, B.,1980), especially for $R<R_0$. This is due to my larger electron-impurity potential. For $R>R_0$. It is in agreement with(Gold, A.and Ghazali, A., 1990) .

The further use of the analytical expressions derived in this work by other researchers will certainly simplify the study of electronic properties of QWWs.

Since I am more interested in simple analytical expressions for the dielectric function, I do not review the situation with computer simulation or numerical work. For developments in this direction the reader is referred to (Tanatar B.,1994) and references there in .

If one tries to improve the analytical solution of the Hartree potential, one runs into problems in the solution of the Schrodinger equation. This is why I have resorted to a perturbation theory type of calculation in the present study.

This matter is further complicated if one includes the exchange-correlation part of the potential into the Schrodinger equation. I may conclude that the most agreeable approach to the self- consistency problem, at present, is a variational calculation which includes a minimum amount of numerical work.

REFERENCES

1. Ando,A .,Fowler, A.B. &Stern,F.,(1989) . Electronic properties of two dimensinal systems . Review of Modern Physics. 54,437
2. Fishman, G. (1986). Mobility in a quasi-one-dimensional semiconductor: An analytical approach. Physical Review B 34, 2394.
3. Gershoni, D., Temkin, H., Dolan, G., Dunsmuir, J., Chu, S. & Panish, M. (1988). Effects of two-dimensional confinement on the optical properties of InGaAs/InP quantum wire structures. Applied physics letters, 53, 995-997 .
4. Gold, A.& Ghazali, A., (1990). Analytical results for semiconductor quantum-well wire: Plasmons, shallow impurity states, and mobility. Physical Review B 41, 7626 .
5. Harrison, P.&Valavanis, A. (2016).Quantum Wells, Wires and Dots: Theoretical and Computational Physics of Semiconductor Nanostructures.
6. Haug, H., & Koch, S. W. (2009). Quantum theory of the optical and electronic properties of semiconductors .
7. Kallin, C., Leavens, C., & Taylor, R. (1988). Interfaces, Quantum Wells, and Superlattices: Plenum, New York.
8. Kodama, T., & Osaka, Y. (1986). Binding energy of a screened hydrogenic impurity in a quasi one-dimensional electron gas. Journal of the Physical Society of Japan, 55, 3941-3947 .
9. Laux, S. E., Frank, D., & Stern, F. (1988). Quasi-one-dimensional electron states in a split-gate GaAs/AlGaAs heterostructure. Surface Science, 196(1-3), 101-106 .
10. Nag, B., & Gangopadhyay, S. (1993). Energy levels in quantum wires with finite barrier potential. physica status solidi (b), 179(2), 463-471 .
11. Notomi, M., Naganuma, M., Nishida, T., Tamamura, T., Iwamura, H., Nojima, S., & Okamoto, M. (1991). Clear energy level shift in ultranarrow InGaAs/InP quantum well wires fabricated by reverse mesa chemical etching. Applied physics letters, 58(7), 720-722 .
12. Nozières, P., & Pines, D. (1999). Theory of quantum liquids: Westview Press.
13. Gradshteyn, S., & IM Ryzhl. (1980). k, Tables of Integrals, series and products: Academic Press, New York.

14. Sarma, S. D. (1984). Quantum size effects on the plasma dispersion in quasi-two-dimensional electron systems. Physical Review B, 29(4), 2334 .
15. Tanatar, B. (1994). Collective modes in a quasi-one-dimensional, two-component electron liquid. Solid state communications, 92(8), 699-702 .
16. Tsuchiya, M., Gaines, J., Yan, R., Simes, R., Holtz, P., Coldren, L., & Petroff, P. (1989). Optical anisotropy in a quantum-well-wire array with two-dimensional quantum confinement. Physical review letters, 62(4), 466 .
17. Weisbuch, C., Vinter, B., & Brennan, K. F. (1992). Quantum semiconductor structures. Physics Today, 45, 88 .
18. Weng, Y., & Leburton, J. (1989). Impurity scattering with semiclassical screening in multiband quasi-one-dimensional systems. Journal of applied physics, 65, 3089-3095.
19. Zhao, X, Wei, CM., L Yang,L., MY Chou,MY. (2004). Quantum confinement and electronic properties of silicon nanowire. - Physical review letters, 92, 236805.
20. Montes, A., Duque,CA (1997).The binding energies of shallow donor impurities in GaAs quantum-well wires under applied electric fields. Journal of applied physics. aip.scitation.org
21. Goni, AR., Pinczuk, A, Weiner,JS Calleja, JM.,Dennis, BS (1991). One-dimensional plasmon dispersion and dispersionless intersubband excitations in GaAs quantum wires . Physical review Lett. 67 3298.
22. Aciksoz, E. Bayrak, O. Soylu,A .(2016). Binding energy of the donor impurities in GaAs—Ga 1− x Al x As quantum well wires with Morse potential in the presence of electric and magnetic fields. Chinese Physics B
23. Burileanu, A., Radu, LM . (2011) THz laser field effect on the optical properties of cylindrical quantum well wires. Optics Communications..

الملخص: هذه دراسة نظرية للخصائص الإلكترونية لنظم شبه موصلة ذات بعد واحد تقريباً. استخدم نهج تحليلي متناسق تقريباً لحساب نطاقات الطاقة الفرعية والكثافة الإلكترونية. تم استعمال إلكترون – شوائب وإلكترون - إلكترون كجهد تفاعلي لنموذج نطاقي الطاقة، لسلك اسطواني كمي. وكانت النتائج التحليلية للإلكترون– شوائب وإلكترون – إلكترون للجهد التفاعلي تتوافق مع نتائج حسابات النموذج. باستعمال النتائج التحليلية تم مناقشة مجالات مختلفة للخواص الإلكترونية للأسلاك شبه الموصلة الكمومية مثل طاقة الربط للشوائب الضحلة، فصل الطيف الفرعي الداخلي، طيف الطاقة المثارة، وسرعة انتقال الإلكترون.

الكلمات المفتاحية: الخواص الإلكترونية، الشوائب الضحلة، أشباه الموصلات، البئر الكمومي لسلك، أنظمة أحادية البعد.