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Constrained Probabilistic multi-source Continuous Review Inventory Model with Dagum and Kumaraswamy-Dagum distributions

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Abstract: The main objective of this paper is to minimize the expected total cost using Lagrange multiplier approach with decreasing varying holding cost for probabilistic continuous review single-item multi-source inventory model under a restriction on the storage space. The optimal order quantity and the optimal reorder point for the best source which achieve the goal when lead time demand follows Dagum and Kumaraswamy-Dagum distributions of obtained. Also, an application is analyzed and reach the goal of minimizing the expected total cost with simulation data.

Keywords: Continuous Review, Lagrange multiplier technique, Storage space, decreasing varying holding cost, Dagum and Kumaraswamy -Dagum distributions.

نموذج المراجعة المستمرة المقيد للمخزون مع توزيعات داجوم وكوماراسوامي – داجوم

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المستخلص: الهدف الرئيسي من هذه الورقة هو تقليل التكلفة الكلية المتوقعة باستخدام طريقة مضاعف لاغرانج للنموذج المراجعة المستمرة للمخزون الاحتمالي لسلعة واحد فقط من عدة مصادر في حال وجود القيد على مساحة التخزين. والايجاد كمية الطلب المثلى ونقطة إعادة الطلب المثلى للمصدر التي تحقق الهدف عندما يتبع الطلب خلال الفتره الزمنيه توزيعات داجوم وكوماراسوامي -داجوم للحصول على أفضل مصدر لسلعة. من خلال تطبيق تم الحصول على الهدف من هذه الدراسة.

الكلمات المفتاحية: مراجعة مستمرة، طريقة مضاعف لاغرانج، مساحة التخزين، تكلفة التخزين المتغيرة تناقصيا، توزيعات داجوم وكوماراسوامي-داجوم.

Introduction.

Inventory system is one of the most important fields of research that has considerable relevance to any sector of the economy. In view of the importance of multiplicity of sources in the inventory, we will discuss a single-item, multi-source (SIMS) inventory system. This system can be found in SIMS procurement and inventory system due to the demand stimulus. In all cases, procurement managers replenish stock in order to meet the demand of the product on a regular basis. Inventory replenishment can be done via procurement from any one of a number of sources under the SIMS. One aspect of the procurement and inventory issue is to select a source that has the lowest total system cost possible. As a part of the SIMS system, procurement and inventory policy will dictate when and how much of a given item should be purchased and from which source. Its concept was developed by (Fabrycky and Banks, 1967) and the application of the concept to the purchase or manufacture decision was presented by (Fabrycky, 1964).

Many authors discussed the probabilistic inventory system models (Abuo-El-Ata et al., 2003). introduced a probabilistic multi-item inventory model with varying order cost and zero lead time under two restrictions by use a geometric programming approach (Fergany & El-Wakeel, 2004). studied the probabilistic single-item inventory problem with varying order cost under two linear constraints (Fergany & Gomaa, 2018). deduced the Probabilistic mixture shortage multi-source inventory model with varying holding cost under constraint (Fergany, 2016). investigated probabilistic multi-item inventory model with varying mixture shortage cost under restrictions (Braglia et al., 2019). examined single product, single-location inventory system continuous review, (Q, r) inventory model for a deteriorating item with random demand and positive lead time with shortages that are allowed and backorders-lost sales mixtures (Fergany & El-Saadani, 2005). studied constrained probabilistic inventory model with the exponential and the Laplace distributions and the varying holding cost which increase in the proposed model by using the Lagrangian multiplier technique. Discussed multi-product, multi-venders inventory models with different cases of the rational function under linear and non-linear constraints via geometric programming approach (El-Wakeel & Al Salman, 2019). Discussed multi-product, multi-venders inventory models with different cases of the rational function under linear and non-linear constraints via geometric programming approach. The case study probabilistic estimates in the application of inventory models for perishable products in SMEs was presented by (Cevallos-Torres & Botto-Tobar, 2019; Fergany & Gawdt, 2011). Studied two different cases of continuous review inventory models with varying increasing holding cost, under service level constraint with mixture shortage when lead time was reduction by the lead time crashing cost.

Recently, Lee (2020) suggested multi-item continuous review inventory (Q, r) model that include a general form of dependence and correlation in demands among components by using a multivariate Gaussian probability distribution. An optimization approach for inventory costs in probabilistic inventory models as a case study was introduced by (Pulido-Rojano et al., 2020; Fergany et al., 2021) discussed scheduling period inventory model with Weibull deteriorating for crisp and fuzzy. When demand during any scheduling time is a random variable and there is no shortage, the deterioration rate follows the Weibull distribution with varying and limited estimated deteriorating cost with two-parameter. Mahapatra et al. (2021) developed two algorithms as a method for obtaining the optimal solution with numerical illustration on a continuous review production inventory system with variable preparation time in a fuzzy random environment.

In this paper, a constrained probabilistic single-item, multi-source (SIMS) continuous review inventory model with decreasing varying holding cost under the expected storage space cost restriction will be investigated. The objective is to determine the reorder point and the order quantity, in the light of system and cost parameters, so that the sum of all costs associated with the system will be minimized. The optimal solutions of the quantity of order (Q), the reorder point (r), which minimize the expected total cost, $E(TC(Q_m,r_m))$, using Lagrange transform, are obtained mathematic on the Dagum distribution and the Kumaraswamy Dagum distribution of the lead time demand. An application is added with its results to observe the optimal source for item.

Research Problem:

Classical probabilistic inventory models were and are still being used extensively in numerous zones that include economy, management science and industrial engineering. There are probabilistic inventory models that need treatment to solve some economic problems. The most important models that have been treated is the probabilistic continuous review. The continuous review inventory model has been addressed for several years where several assumptions and conditions are represented in models in many research papers in which authors analyzed. This paper provides a new model of single item multi sources probabilistic continuous review mixture shortage with varying decreasing holding cost inventory model with assumption for developing model. In this paper provides a new model of single item multi sources probabilistic continuous review mixture shortage with varying decreasing holding cost inventory model.

Notations and Model Development:

In this section, the notations for the model development are defined as follows:

$\bar{D}$ = The average demand.

X = The lead time demand.

Q_m = The decision variable representing the order quantity per cycle for the single-item, multi-source.

Q_m^*= The optimal value of the order quantity per cycle for the single-item, multi-source.

r_m = The decision variable representing the reorder point per for the single-item, multi-source.

r_m^*= The optimal reorder point per cycle for the single-item, multi-source.

C_{om}= The order cost per cycle for the single-item, multi-source.

C_{hm} = The holding cost per unit per cycle for the single-item, multi-source.

C_s = The shortage cost per cycle for the single-item, multi-source.

C_b = The backorder cost per unit per cycle for the single-item, multi-source.

C_l = The lost sales cost per unit per cycle for the single-item, multi-source.

$E(OC_m)$ = The expected order cost for the single-item, multi-source.

$E(HC_m)$ = The expected holding cost for the single-item, multi-source.

$E(SC)$ = The expected shortage cost for the single-item, multi-source.

$E(BC)$ = The expected backorder cost for the single-item, multi-source.

$E(LC)$ = The expected lost sales cost for the single-item, multi-source.

$C_{hm}(Q_m)$ = The varying holding cost per unit per cycle for the single-item, multi-source.

WQ_m = The storage space cost for the single-item, multi-source.

$E(TC(Q_m, r_m))$ = The expected total cost for the single-item, multi-source.

$min\ E\ (TC_m)$ = The minimum expected total cost for the single-item, multi-source.

K =The limitation on the storage space cost.

λ_m =Lagrange multiplier for the single-item, multi-source.

λ_m^* = The optimal value of the Lagrange multiplier for the single-item, multi-source.

The Mathematical Model:

In this section, we will present the continuous review model with decreasing varying holding cost according to the assumptions on SIMS system. The distribution of the lead time demand (X) depends on the distribution of the demand when the demand (D) is variable and the lead time (L) is constant. It is probable to develop the expected total cost which consists of the sum of the expected order cost $E_m(OC)$, the expected varying holding cost $E_m\big(HC(Q_m)\big)$, and the expected mixture shortage cost $E_m(BC)$ and $E_m(LC)$ which are given in the following equations:

$$\boldsymbol{E\big(TC(Q_m, r_m)\big) = E(TC_m) = \sum_{m=1}^{n}\big[E_m(OC) + E_m\big(HC_m(Q_m)\big) + E_m(SC)\big],} \quad (1)$$

where

$$\boldsymbol{E_m(OC) = c_{om}\frac{\bar{D}}{Q_m},}$$

$$\boldsymbol{E_m(HC(Q_m)) = c_{hm}(Q_m)^{-\beta}(\frac{Q_m}{2} + r_m - E(\chi) + (1-\gamma)\bar{S}(r_m),}$$

$$\boldsymbol{E_m(SC) = E_m(BC) + E_m(LC),}$$

$$\boldsymbol{E_m(BC) = c_b\gamma\left(\frac{\bar{D}}{Q_m}\right)\bar{S}(r_m) = c_b\gamma\left(\frac{\bar{D}}{Q_m}\right)\int_{r_m}^{\infty}(\chi_m - r_m)d\chi_m,}$$

and

$$\boldsymbol{E_m(LC) = c_l(1-\gamma)\left(\frac{\bar{D}}{Q_m}\right)\bar{S}(r_m) = c_l(1-\gamma)\left(\frac{\bar{D}}{Q_m}\right)\int_{r_m}^{\infty}(\chi_m - r_m)d\chi_m}$$

By substituting varying holding cost, shortage cost and order cost in (1), we have:,

$$E\big(TC(Q_m,r_m)\big)=\sum_{m=1}^{n}\left[c_{om}\frac{\bar{D}}{Q_m}+c_{hm}(Q_m)^{-\beta}\left(\frac{Q_m}{2}+r_m-E(\chi)+(1-\gamma)\bar{S}(r_m)\right)+c_b\gamma\left(\frac{\bar{D}}{Q_m}\right)\bar{S}(r_m)+c_l(1-\gamma)\left(\frac{\bar{D}}{Q_m}\right)\bar{S}(r_m)\right] \qquad (2)$$

Subject to the following the storage space cost constraint:

$$\sum_{m=1}^{n}[WQ_m\le k]. \qquad (3)$$

By solving equation (2), under the above restrictions, in order to minimize the expected total cost $E(TC(Q_m,r_m))$ using Lagrange multipliers technique we get:

$$G=\sum_{m=1}^{n}[E(TC_m)+\lambda_m(WQ_m-k)]$$

$$=\sum_{m=1}^{n}\left[\frac{c_{om}\bar{D}}{Q_m}+c_{hm}Q_m^{-\beta}\left[\frac{Q_m}{2}+r_m-E(\chi)\right]+\frac{c_b\bar{D}}{Q_m}\gamma\bar{S}(r_m)+\frac{c_l\bar{D}}{Q_m}(1-\gamma)\bar{S}(r_m)+\lambda_m[WQ_m-k_m]\right]. \text{ (4)}$$

The optimal values of the order quantity (Q_m^*) and reorder point (r_m^*) can be calculated, by setting each of the corresponding first partial derivatives of the above equation, with respect to zero, which are minimizing the expected total cost, then the following equations can be obtained as follows:

$$\frac{\partial G}{\partial Q_m}=-\frac{c_{om}\bar{D}}{Q_m^2}-\beta Q_m^{-(1+\beta)}c_{hm}\left(\frac{Q_m}{2}+r_m-E(\chi)+(1-\gamma)\bar{S}(r_m)\right)+\frac{c_{hmQ_m^{-\beta}}}{2}-\frac{c_b\bar{D}}{Q_m^2}\gamma\bar{S}(r_m)-\frac{c_l\bar{D}}{Q_m^2}(1-\gamma)\bar{S}(r_m)+\lambda_mW=0$$

Therefore,

$$-2\beta c_{hm}Q_m^{1-\beta}[r_m-E(\chi)+(1-\gamma)\bar{S}(r_m)]+c_{hm}Q_m^{2-\beta}-2c_{om}\bar{D}-2c_b\bar{D}\gamma\bar{S}(r_m)-2c_l\bar{D}(1-\gamma)\bar{S}(r_m)+2\lambda_mWQ_m^2=0,$$

which implies

$$(1-\beta)c_{hm}Q_m^{2-\beta}-\beta c_{hm}Q_m^{1-\beta}[r_m-E(\chi)+(1-\gamma)\bar{S}(r_m)]-M-2B\bar{S}(r_m)+2\lambda_mWQ_m^2=0, \qquad (5)$$

where

$M=2c_{om}\bar{D}$,

$B=c_b\bar{D}\gamma+c_l\bar{D}(1-\gamma)$,

and

$$\frac{\partial G}{\partial r_m}=c_{hm}Q_m^{1-\beta}\big(1-(1-\gamma)R(r_m)\big)-\frac{c_b\bar{D}\gamma}{Q_m}R(r_m)-\frac{c_l\bar{D}(1-\gamma)}{Q_m}R(r_m)=0,$$

Hence,

$$c_{hm}Q_m^{1-\beta} - c_{hm}Q_m^{1-\beta}(1-\gamma)R(r_m) - c_b\bar{D}\gamma R(r_m) - c_l\bar{D}(1-\gamma)R(r_m) = 0.$$

Thus,

$$R(r_m) = \frac{c_{hm}Q_m^{1-\beta}}{c_{hm}Q_m^{1-\beta}(1-\gamma)+B} \ . \qquad (6)$$

It is obvious that we cann ot find the solution for equations (5) and (6). Thus, we resort to algorithms and iterative relationships to find minimum expected total cost.

Algorithm

Step 1: Calculate the first order quantity(Q_1) by assuming two values *and* λ_m, β. Suppose the initialvalue $E(x) = r_m$ and $\bar{S}(r_0) = 0$. Input all values in the inventory model during the application.

Step 2: Calculate value of r_1 by $R(r)$ of the used distribution and hence, we deduce $\bar{S}(r_1)$.

Step 3: To calculate a new order quantity use the value of Q_2 to find r_2 by use $\bar{S}(r_1)$ and r_1 of the distribution as in the step 2. Repeat the steps till get $r_m = r_{m+1}$ and Q_m=Q_{m+1} .

Step 4: Derive the minimum expected total cost E (TC) and the storage space cost by using the lastoptimal values of r_m and Q_m.

Step 5: Check the restriction. $WQ_m \leq k$, then record the values r_m and Q_m as the optimal values r_m^* and Q_m^* which minimize the expected total cost at this value of β *under* the constraint if not, go to step 6.

Step 6: If $WQ_m > k$, go to step 1, and alteration the value of λ_m. Repeat all the steps till therestrictions hold as in table 6.

Step 7: Repeat each procedure to calculate the optimal values which minimize expected total costwhen we change the values of β *to* another values.

The Model with income distributions:

Suppose the lead time demand follows Dagum and Kumaraswamy-Dagum distributions.

The Model with Dagum distribution:

Suppose that the lead time demand follows the Dagum distribution with three parameters , φ and η. The reliability function is given by:

$$R(r) = \int_r^{\infty} f(\chi)d\chi = 1 - (1+\delta r^{-\varphi})^{-\eta}, \quad \chi \geq 0 \quad , \delta, \eta, \varphi > 0. \qquad (7)$$

Where the density function is given by:

$$f(\chi) = \delta\varphi\eta\chi^{-\varphi-1}(1+\delta\chi^{-\varphi})^{-\eta-1}$$

The expected shortage quantity is defined as:

$$\bar{S}(r_m) = \int_r^\infty (\chi - r) f(\chi) d\chi.$$

Using the following expansion

$$(1-u)^{-\frac{1}{\varphi}} = \binom{\frac{1}{\varphi}+i-1}{i} u^i$$

the expected shortage quantity becomes:

$$\bar{S}(r) = A[1-(1+\delta r^{-\varphi})^{-\eta(i+1)+1}] - r[1-(1+\delta r^{-\varphi})^{-\eta}], \qquad (8)$$

Where

$$A = \delta^{\frac{1}{\varphi}} \frac{\sum_{i=0}^{\infty} \binom{1/\varphi + i - 1}{i}}{-\eta(i+1)+1}$$

By inserting (7) and (8) in (6) and (5) for any source, the projected total cost of equation (2) can be minimized analytically. When the lead time demand follows the Dagum distribution, the ideal values r_m and Q_m found to be:

$$R(r_m) = [1-(1+\delta r^{-\varphi})^{-\eta}] - \frac{c_{hm} Q_m^{1-\beta}}{c_{hm} Q_m^{1-\beta}(1-\gamma) + B} \qquad (9)$$

$$(1-\beta)c_{hm}Q_m^{2-\beta} - \beta c_{hm} Q_m^{1-\beta}[r_m - E(\chi) + (1-\gamma)[A(1-(1+\delta r^{-\varphi})^{-\eta(i+1)+1}) - r(1-(1+\delta r^{-\varphi})^{-\eta})]] + 2\lambda_m g Q_m^2 - M - 2B[A(1-(1+\delta r^{-\varphi})^{-\eta(i+1)+1}) - r(1-(1+\delta r^{-\varphi})^{-\eta})] = 0 \qquad (10)$$

The Model with Kumaraswamy-Dagum distribution

If the lead time demand follows the Kumaraswamy-Dagum distribution with α, , δ, φ and η parameters, the density function is given by:

$$f(\chi) = \eta\delta\,\varphi\,\alpha\,\theta\chi^{-\varphi-1}(1+\delta\chi^{-\varphi})^{-\eta\alpha-1}[1-(1+\delta\chi^{-\varphi})^{-\eta\alpha}]^{\theta-1}, \chi > 0 \quad , \alpha, \varphi, \eta, \delta, \theta > 0$$

The reliability function can be obtained as:

$$R(r) = [1-(1+\delta r^{-\varphi})^{-\eta\alpha-1}]^{\theta}, \qquad (11)$$

and the expected shortage quantity is given by:

$$\bar{S}(r) = \int_r^\infty (\chi - r) d\chi,$$

By using the following expansions:

$$(1-u)^{\theta-1}=\sum_{j=1}^{\infty}\frac{(-1)^{j}\Gamma(\theta)}{\Gamma(\theta-j)j!}u^{j},$$

And

$$(1-u^{\frac{1}{\alpha\eta}})^{-\frac{1}{\varphi}}=\sum_{k=1}^{\infty}\binom{\frac{1}{\varphi}+k-1}{k}u^{\frac{k}{\alpha\eta}},$$

we obtain

$$\bar{S}(r)=C[1-\sum_{j,k=0}^{\infty}(1+\delta r^{-\varphi})^{-\eta\alpha(j+1)-k+1}]-r[1-(1+\delta r^{-\varphi})^{-\eta\alpha}]^{\theta}\,,\quad (12)$$

Where

$$C=\sum_{j,k=1}^{\infty}\frac{\delta^{\frac{1}{\varphi}}(-1)^{j}\Gamma(\theta)\Gamma(\frac{1}{\varphi}+K)}{(\eta\alpha(j+1)+K-1)\Gamma(\theta-j)\Gamma(j+1)\Gamma(\frac{1}{\varphi}-1)\Gamma(k+1)}.$$

By inserting from (11) and (12) into (6) and (5) for any source, the projected total cost of equation (2) can be minimized analytically. When the lead time demand follows the Dagum distribution, the ideal values rm and Qm found to be:

$$R(r_m)=[1-(1+\delta r^{-\varphi})^{-\eta\alpha}]^{\theta}-\frac{c_{hm}Q_m^{1-\beta}}{c_{hm}Q_m^{1-\beta}(1-\gamma)+B},\quad (13)$$

$$(1-\beta)c_{hm}Q_m^{1-\beta}-\beta c_{hm}Q_m^{1-\beta}[r_m-E(\chi)+(1-\gamma)[C(1-\sum_{j,k=0}^{\infty}(1+\delta r^{-\varphi})^{-\alpha\eta(j+1)-k+1})-r(1-(1+\delta r^{-\varphi})^{-\alpha\eta})^{\theta}]]+2\lambda_m g Q_m^2-M-2B[C(1-\sum_{j,k=0}^{\infty}(1+\delta r^{-\varphi})^{-\alpha\eta(j+1)-k+1})-r(1-(1+\delta r^{-\varphi})^{-\alpha\eta})^{\theta}]=0\,.\quad (14)$$

A linear cost function, which consists of constant expected ordering cost, a expected variable holding cost proportional to order size and expected shortage cost, is used in some optimization investigations. We arrived at the order quantity and reorder point equations but cannot find the exact solution for equations. To get the solution we use algorithms. We presented findings regardinc minimum the previa expected total cost for the unit when lead time demand follows Dagum and Kumaraswamy-Dagum distributions.

Simulation Study:

In this section, we examine the performance of Dagum and Kumaraswamy-Dagum distributions with 1000 iterations by using the inverse transformation method for Maximum Likelihood Estimation. We consider the parameters (η = 1.25, δ = 1.5, = 4) for Dagum distribution and (η = 2.8, δ = 0.01, = 0.1, α = 0.19, θ = 1) for Kumaraswamy-Dagum distribution with sample sizes n = 200, 500, 1000, 2000 and 5000. Tow accuracy measures which are the mean-squared error (MSE) and the bias are calculated to evaluate the estimates of the parameters. It can be observed that when the sample size increases, the bias and MSE a decrease.

Table (1) The Estimate, Bias and MSE of Maximum Likelihood Estimation (MLE) with different sample sizes

			Dagum			Kumaraswamy-Dagum	
n	**Parameters**	**Estimate**	**MSE**	**Bias**	**Estimate**	**MSE**	**Bias**
200	η	1.391755	0.3486254	0.14175528	2.8757836	0.01410457	0.07578357
	δ	1.646477	0.9396225	0.14647745	0.4083073	0.17533281	0.39830727
	φ	4.032693	0.2111365	0.03269304	0.9904340	0.89252510	0.89043404
	α	-	-	-	0.2764940	0.00880779	0.08649396
	θ	-	-	-	1.1831092	0.07942643	0.18310924
500	η	1.298320	0.06704178	0.04831994	2.8486188	0.002972252	0.04861883
	δ	1.544617	0.23369155	0.04461678	0.3268447	0.108215755	0.31684468
	φ	4.012970	0.07388151	0.01296978	0.7811198	0.502146526	0.68111975
	α	-	-	-	0.2527391	0.004506864	0.06273905
	θ	-	-	-	1.1195293	0.017432557	0.11952930
1000	η	1.259403	0.02719528	0.009402972	2.8416867	0.002099982	0.04168668
	δ	1.553720	0.12335007	0.053720398	0.2813472	0.075563754	0.27134721
	φ	4.016611	0.03641489	0.016610911	0.6706833	0.346627195	0.57068326
	α	-	-	-	0.240191	0.002674295	0.05019165
	θ	-	-	-	1.1049982	0.013290032	0.10499816
2000	η	1.257400	0.01368677	0.0073998399	2.8320272	0.001692859	0.03202722
	δ	1.516408	0.05116604	0.0164082216	0.2594175	0.062927809	0.24941754
	φ	4.000788	0.01775430	0.0007878212	0.5867688	0.249261564	0.48676880
	α	-	-	-	0.2336373	0.001972685	0.04363731
	θ	-	-	-	1.0811509	0.011153429	0.08115094
5000	η	1.255543	0.004971281	0.005542918	2.8215675	0.0005417188	0.02156750
	δ	1.500325	0.020240656	0.0003253234	0.2400615	0.0534136870	0.23006150
	φ	3.999962	0.007158343	0.00003824349	0.5067825	0.1725898364	0.40678254
	α	-	-	-	0.2277892	0.0014703209	0.03778922
	θ	-	-	-	1.0544219	0.0034700422	0.05442188

Application:

An electronic company manager in Egypt decided to order one electronic appliance according to model assumptions from three different companies. He wishes to get an optimal policy to minimize the expected total cost when the storage space equal 100 Square meters. The parameters for a single item and multi-source are given in Table 2 and Table 3 respectively.

Table (2) the parameters for a single item.

D	cb	cl	n	γ	K	W
300	20	30	200	0.7	14.5	0.5

Table (3) the order cost and holding cost for multi-source

Cost	Source 1	Source 2	Source 3
Com	20	25	24
Chm	10	9	9.5

Simulation studies are conducted using the simple size n=200 and constant number β from 0.1 to 0.6 for Dagum and Kumaraswamy-Dagum distrubtions. Equations 9 and 10 applied for the Dagum distribution as well as 13 and 14 for the Kumaraswamy-Dagum distribution using tables 2 and 3 to find the minimum expected total cost for each source and the ideal solutions r* and Q*. Tables 4 and 5 show the results of different constant numbers assuming different values of the parameter β. The best minimum expected total costs for source when β = 0.6 for Dagum and Kumaraswamy-Dagum distrubtions are also displayed in table 7.

Table (4) the result of Dagum distribution

	Source 1				Source 2				Source 3			
β	λ1	r1	Q1	min(E(TC1))	λ2	r2	Q2	min(E(TC2))	λ3	r3	Q3	min(E(TC3))
0.1	5.7	2.3967	28.99	19.1668	10.07	2.46134	28.99	20.5915	8.95	2.42744	28.99	20.4106
0.2	8.56	2.6120	28.98	18.1284	12.64	2.68228	28.99	19.6283	11.67	2.64576	28.99	19.4133
0.3	10.47	2.8451	28.99	17.3504	14.39	2.92203	28.99	18.9473	13.5	2.88222	28.99	18.6843
0.4	11.77	3.0984	28.99	16.8067	15.56	3.18188	28.99	18.4458	14.73	3.13867	28.99	18.1558
0.5	12.63	3.3731	28.99	16.3995	16.35	3.46396	28.99	18.0883	15.56	3.41707	28.99	17.7791
0.6	13.21	3.671	28.99	16.1121	16.6	3.7703	28.99	17.828	15.83	3.7194	28.99	17.5067

Table (5) the result of Kumaraswamy-Dagum distribution

	Source 1				Source 2				Source 3			
β	λ1	r1	Q1	min(E(TC1))	λ2	r2	Q2	min(E(TC2))	λ3	r3	Q3	min(E(TC3))
0.1	7.83	2.173*10−8	28.99	18.9196	12.03	6.441*10−8	28.99	20.3399	11	3.688 *10−8	28.99	20.164
0.2	10.15	6.943*10−7	28.99	17.8897	14.12	2.043*10−6	28.99	19.4218	13.21	1.175 *10−6	28.99	19.1954
0.3	11.65	0.000022	28.99	17.1659	15.46	0.000063	28.99	18.7701	14.62	0.000036	28.99	18.497
0.4	12.58	0.000659	28.99	16.6461	16.26	0.001916	28.99	18.3006	15.5	0.0011	28.99	18.0073
0.5	13.12	0.01982	28.99	16.2756	16.74	0.05734	28.99	17.9645	16	0.03324	28.99	17.6533
0.6	13.38	0.56042	28.99	16.0403	16.92	1.7017	28.99	17.7967	16.22	0.9875	28.99	17.4418

Table 6: The value of * for the first source at β = 0.6

Λ	WQm	min(E(TC))
0	151.621	0.302426
0.5	135.144	1.05025

Λ	WQm	min(E(TC))
5	23.3419	6.41911
13.3	14.5388	15.9453
13.36	14.5067	16.0141
13.37	14.5014	16.0255
13.38	14.495	16.0403
13.39	14.4908	16.047

Table 7: The optimal and minimum expected total cost at $\beta = 0.6$

Distribution	*	r*	Q*	(E(TC))	Source
Dagum	13.21	3.6716	28.99	16.1121	1
Kumaraswamy-Dagum	13.38	0.56042	28.99	16.0403	1

Conclusion.

In this paper, we discussed constrained multi-source probabilistic continuous review inventory models with decreasing varying holding cost. When the lead time demand follows the Dagum and the Kumaraswamy-Dagum distributions, we obtained the minimum expected total cost by using Lagrangian multiplier technique. From the results that we got form the distributions, we found that the best value for minimum expected total cost at $\beta = 0.6$ as shown in Table 6. Also, the Kumaraswamy-Dagum distribution is better slightly than Dagum distribution and the first source is the best source. We got the results by using Mathematica program to obtained the best source and R program to simulate data for distributions.

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